MATHEMATICS TEXTBOOK

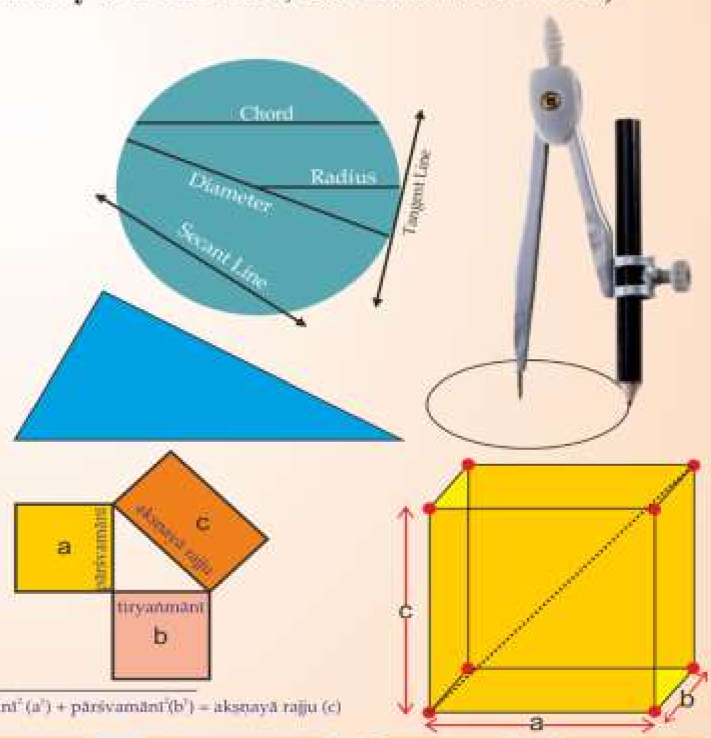
(On the basis of vedic Mathematics and Sanskrit Literature of Mathematics)

Veda Bhushan IV Year / Purva Madhyama - I Year / Class IX

MAHARSHI SANDIPANI RASHTRIYA VEDA SANSKRIT SHIKSHA BOARD

(Established and Recognized by the Ministry of Education, Government of India)

प्रमाणं तृतीयेन वर्धयेत्तच्च चतुर्थेनात्मचतुस्त्रिंशोऽनेन सविशेषः ।।
योगं करण्योर्महतीं प्रकल्प्य वधस्य मूलं द्विगुणं लघुं च ।
योगान्तरे रूपवदेतयोः स्तो वर्गेण वर्गं गुणयेद्भजेच्च ॥
भाज्याच्छेदः शुद्ध्यति प्रच्युतः सन् स्वेषु स्वेषु स्थानकेषु क्रमेण ।
यैर्यैर्वर्णैः संगुणो यैश्च रूपैर्भागाहारे लब्धयस्ताः स्युरत्र ॥
एकाव्यक्तं शोधयेद् अन्यपक्षाद् रूपाण्यन्यस्येतरस्माच्च पक्षात् ।
शेषाव्यक्तेनो उद्धरेद् रूपशेषं व्यक्तं मानं जायतेऽव्यक्तराशेः।।
अव्यक्तानां द्व्यादिकानामपीह ,यावत्तावद् द्व्यादिनिघ्नं हृतं वा।
युक्तोनं वा कल्पयेद् आत्मबुद्ध्या मानं क्वापि व्यक्तमेवं विदित्वा।
तिरश्चीनो विततो रश्मिरेषामधः स्विदासीऽदुपरि स्विदासीऽत् ।
रेतोधाऽआसन्महिमानऽआसन्त्स्वधाऽअवस्तात्प्रयतिः परस्तात् ॥
दीर्घचतुरश्रस्याक्ष्णयारज्जुः पार्श्वमानी तिर्यङ्मानी च
यत्पृथग् भूते कुरुतस्तदुभयं करोति ।
यो अक्रन्दयत् सलिलं महित्वा योनिं कृत्वा त्रिभुजं शयानः।
वत्स कामदुघो विराजः स गुहा चक्रे तन्वः पराचैः।
सर्वदोर्युतिदलं चतुःस्थितं बाहुभिर्विरहितं च तद्वधात्।
मूलमस्फुटफलं चतुर्भुजे स्पष्टमेवमुदितं त्रिबाहुके।।
क्षेत्रफलं वेधगुणं खाते घनहस्तसंख्या स्यात्।



ekādhikena pūrveṇa | ekanyūnena pūrveṇa | vinakulam | vilokanam

ānurūpyeṇa | bījāṅka | nikhilaṃ navataścaramaṃ daśataḥ

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PREFACE

(In the light of NEP 2020)

The Ministry of Education (Department of Higher Education), Government of India established Rashtriya Veda Vidya Pratishthan in Delhi under the Chairmanship of Hon'ble Education Minister (then Minister of Human Resource Development) under the Societies Registration Act, 1860 (XXI of 1860) on 20th January, 1987. The Government of India notified the resolution in the Gazette of India vide no 6-3/85- SKT-IV dated 30-3-1987 for establishment of the Pratishthan for preservation, conservation, propagation and development of oral tradition of Vedic studies (Veda Samhita, Padapatha to Ghanapatha,Vedanga, Veda Bhashya etc), recitation and intonation of Vedas etc and interpretation of Vedas in scientific lines. In the year 1993 the name of the organization was changed to Maharshi Sandipani Rashtriya Veda Vidya Pratishthan (MSRVVP) and it was shifted to Ujjain, Madhya Pradesh.

The National Education Policy of 1986 and Revised Policy Formulations of 1992 and also Programme of Action (PoA) 1992 have mandated Rashtriya Veda Vidya Pratishthan for promoting Vedic education throughout the country. The importance of India's ancient fund of knowledge, oral tradition and employing traditional Guru's for oral education was also emphasized in the PoA.

In accordance with the aspirations of the nation, national consensus and policy in favour of establishing a Board for Veda and Sanskrit Education at national level, the General Body and the Governing Council of MSRVVP under the Chairmanship of Hon'ble Education Minister, Government of India, have set up "Maharshi Sandipani Rashtriya Veda Sanskrit Shiksha

Board" (MSRVSSB) in tune with the mandate of the Pratishthan and its implementation strategies. The Board is necessary for the fulfillment of the objectives of MSRVVP as envisioned in the MoA and Rules. The Board has been approved by the Ministry of Education, Government of India and recognized by the Association of Indian Universities, New Delhi. The bye-laws of the Board have been vetted by Central Board of Secondary Education and curriculum structure have been concurred by the National Council of Educational Research and Training, New Delhi.

It may also be mentioned here that the committee "Vision and Roadmap for the Development of Sanskrit - Ten year perspective Plan", under the Chairmanship of Shri N. Gopalaswamy, former CEC, constituted by the Ministry of Education Govt. of India in 2015 recommended for establishment of a Board of Examination for standardization, affiliation, examination, recognition, authentication of Veda Sanskrit education up to the secondary school level. The committee was of the opinion that the primary level of Vedic and Sanskrit studies should be inspiring, motivating and joyful. It is also desirable to include subjects of modern education into Vedic and Sanskrit Pathashalas in a balanced manner. The course content of these Pathashalas should be designed to suit to the needs of the contemporary society and also for finding solutions to modern problems by reinventing ancient knowledge.

With regard to Veda Pathashala-s it is felt that they need further standardization of recitation skills along with introduction of graded materials of Sanskrit and modern subjects so that the students can ultimately acquire the capabilities of studying Veda bhashya-s and mainstreaming of students is achieved for their further studies. Due emphasis may also be given for the study of Vikriti Patha of Vedas at an appropriate level. The members of the committee have also expressed their concern that the Vedic recitation studies are not uniformly spread all over India; therefore, due steps may be taken to improve the situation without in anyway interfering with regional variations of recitation styles and teaching method of Vedic recitation.

It was also felt that since Veda and Sanskrit are inseparable and complementary to each other and since the recognition and affiliation problems are same for all the Veda Pathashalas and Sanskrit Pathashalas throughout the country, a Board may be constituted for both together. The committee observed that the examinations conducted by the Board should have legally valid recognition enjoying parity with modern Board system of education. The committee observed that the Maharshi Sandipani Rashtriya Veda Vidya Pratishthan, Ujjain may be given the status of Board of Examinations with the name "Maharshi Sandipani Rashtriya Veda Sanskrita Vidya Parishat with headquarters in Ujjain which will continue all programs and activities which were being conducted hitherto in addition to being a Board of Examinations.

The promotion of Vedic education is for a comprehensive study of India's glorious knowledge tradition and encompasses multi-layered oral tradition of Vedic Studies (Veda Samhita, Padapatha to Ghanapatha,Vedanga, Veda Bhashy aetc), recitation and intonation, and Sanskrit knowledge system content. In view of the policy of mainstreaming of traditional students and on the basis of national consensus among the policy making bodies focusing on Vedic education, the scheme of study of Veda stretching up to seven years in Pratishthan also entails study of various other modern subjects such as Sanskrit, English, Mathematics, Social Science, Science, Computer Science, Philosophy, Yoga, Vedic Agriculture, etc. as per the syllabus and availability of time. In view of NEP 2020, this scheme of study is with appropriate inputs of Vedic knowledge and drawing the parallels of modern knowledge in curriculum content focusing on Indian Knowledge System.

In Veda Pathashala-s, GSP Units and Gurukula-s of MSRVVP, affiliated to the Board transact the curriculum primarily based on oral tradition of a particular complete Veda Shakha with perfect intonation and memorization, with additional subsidiary modern subjects such as English, Sanskrit, Mathematics, Science, Social Science and SUPW. Gradually, the Veda Pathashala-s will also introduce other skill and vocational subjects as per their resources.

It is a well-known fact that there were 1131 shakha-s or recensions of Vedas; namely 21 in Rigveda, 101in Yajurveda, 1000 in Samaveda and 9 in Atharva Veda. In course of time, a large number of these shakhas became extinct and presently only 10 Shakhas, namely, one in Rigveda, 4 in Yajurveda, 3 in Samaveda and 2 in Atharvaveda are existing in recitation form on which Indian Knowledge System is founded now. Even in regard to these 10 Shakhas, there are very few representative Vedapathis who are continuing the oral Vedic tradition/ Veda recitation/Veda knowledge tradition in its pristine and complete form. Unless there is a full focus for Vedic learning as per oral tradition, the system will vanish in near future. These aspects of Oral Vedic studies are neither taught nor included in the syllabus of any modern system of school education, nor do the schools/Boards have the systemic expertise to incorporate and conduct them in the conventional modern schools.

The Vedic students who learn oral tradition/ recitation of Veda are there in their homes in remote villages, in serene and idyllic locations, in Veda Gurukulas, (GSP Units), in Veda Pathashala-s, in Vedic Ashrams etc. and their effort for Veda study stretches to around 1900 – 2100 hours per year; which is double the time of other conventional school Board's learning system. Vedic students have to have complete Veda by-heart and recite verbatim with intonation *(udatta, anudatta, swaritaetc),*on the strength of memory and guru parampara, without looking at any book/pothi. Because of unique ways of chanting the Veda mantras, unbroken oral transmission of Vedas and its practices, this has received the recognition in the UNESCO-World Oral Heritage in the list of Intangible Cultural Heritage of Humanity. Therefore, due emphasis is required to be given to maintain the pristine and complete integrity of the centuries old Vedic Education (oral tradition/ recitation/ Veda knowledge Tradition). Keeping this aspect in view the MSRVVP and the Board have adopted unique type of Veda curriculum with modern subjects like Sanskrit, English, Vernacular language, Mathematics, Social Science, Science, Computer Science, Philosophy, Yoga, Vedic Agriculture etc. as well as skill and vocational subjects as prescribed by NEP 2020.

As per Vedic philosophy, any person can become happy if he or she learns both *Para-Vidya and Apara-Vidya.* The materialistic knowledge from the Vedas, their auxiliary branches and subjects of material interest were called *Apara-Vidya*. The knowledge of supreme reality, the ultimate quest from Vedas, Upanishads is called *Para-Vidya.* In all the total number of subjects to be studied as part of Veda and its auxiliaries are fourteen. There are fourteen branches of learning or *Vidyas* - four Vedas, Six Vedangas, Mimamsa (Purva Mimamsa and Uttara Mimamsa), Nyaya, Puranas and Dharma shastra. These fourteen along with Ayurveda, Dhanurveda, Gandharvaveda and Arthashastra become eighteen subjects for learning. All curriculum transaction was in Sanskrit language, as Sanskrit was the spoken language for a long time in this sub-continent.

Eighteen Shilpa-s or industrial and technical arts and crafts were mentioned with regard to the Shala at Takshashila. The following 18 skills/Vocational subjects are reported to be subjects of the study– (1) Vocal music (2) Instrumental music (3) Dancing (4) Painting (5) Mathematics (6) Accountancy (7) Engineering (8) Sculpture (9) Cattle breeding (10) Commerce (11) Medicine (12) Agriculture (13) Conveyancing and law (14) Administrative training (15) Archery and Military art (16) Magic (17) Snake charming (18) Art of finding hidden treasures.

For technical education in the above mentioned arts and crafts an apprenticeship system was developed in ancient India. As per the Upanishadic vision, the vidya and avidya make a person perfect to lead contented life here and liberation here-after.

Indian civilization has a strong tradition of learning of shastra-s, science and technology. Ancient India was a land of sages and seers as well as of scholars and scientists. Research has shown that India had been a Vishwa Guru, contributing to the field of learning (vidya-spiritual knowledge and avidya- materialistic knowledge) and learning centers like modern universities were set up. Many science and technology based advancements of that time, learning methodologies, theories and techniques discovered by the ancient sages have created and strengthened the fundamentals of our knowledge on many aspects, may it be on astronomy, physics, chemistry, mathematics, medicine, technology, phonetics, grammar etc. This needs to be essentially understood by every Indian to be proud citizen of this great country!

The idea of India like "Vasudhaiva Kutumbakam" quoted at the entrance of the Parliament of India and many Veda Mantra-s quoted by constitutional authorities on various occasions are understood only on study of the Vedas and true inspiration can be drawn only by pondering over them. The inherent equality of all beings as embodiment of "sat, chit, ananda" has been emphasized in the Vedas and throughout the Vedic literature.

Many scholars have emphasized that Veda-s are also a source of scientific knowledge and we have to look into Vedas and other scriptural sources of India for the solution of modern problems, which the whole world is facing now. Unless students are taught the recitation of Vedas, knowledge content of Vedas and Vedic philosophy as an embodiment of spiritual and scientific knowledge, it is not possible to spread the message of Vedas to fulfill the aspiration of modern India.

The teaching of Veda (Vedic oral tradition/ Veda recitation/ Veda knowledge Tradition) is neither only religious education nor only religious instruction. It will be unreasonable to say that Vedic study is only a religious instruction. Veda-s are not religious texts only and they do not contain only religious tenets; they are the corpus of pure knowledge which are most useful to humanity as whole. Hence, instruction or education in Veda-s cannot be construed as only "religious education/religious instruction."

Terming "teaching of Veda as a religious education" is not in consonance with the judgment of the Hon'ble Supreme Court (AIR 2013: 15 SCC 677), in Civil Appeal no. 6736 of 2004 (Date of judgment-3rd July 2013). The Vedas are not only religious texts, but they also contain the knowledge in the disciplines of mathematics, astronomy, meteorology, chemistry, hydraulics, physics, science and technology, agriculture, philosophy, yoga, education, poetics, grammar, linguistics etc. which has been brought out in the judgment by the Hon'ble Supreme Court of India.

Vedic education through establishment of Board in compliance with NEP-2020

The National Education Policy-2020 firmly recognizes the Indian Knowledge Systems (also known as 'Sanskrit Knowledge Systems'), their importance and their inclusion in the curriculum, and the flexible approach in combining various subjects. Arts' and Humanities' students will also learn science; try to acquire vocational subjects and soft skills. India's special heritage in the arts, sciences and other fields will be helpful in moving towards multi-disciplinary education. The policy has been formulated to combine and draw inspiration from India's rich, ancient and modern culture and knowledge systems and traditions. The importance, relevance and beauty of India's classical languages and literature is also very important for a meaningful understanding the national aspiration. Sanskrit, being an important modern language mentioned in the Eighth Schedule of Indian Constitution, its classical literature that is greater in volume than that of Latin and Greek put together, contains vast treasures of mathematics, philosophy, grammar, music, politics, medicine, architecture, metallurgy, drama, poetry, storytelling, and more (known as 'Sanskrit Knowledge Systems').These rich Sanskrit Knowledge System legacies for world heritage should not only be nurtured and preserved for posterity but also enhanced through research and put in to use in our education system, curriculum and put to new uses. All of these literatures have been composed over thousands of years by people from all walks of life, with a wide range of socio-economic background and vibrant philosophy. Sanskrit will be taught in engaging and experiential as well as contemporary relevant methods. The use of Sanskrit knowledge system is exclusively through listening to sound and pronunciation. Sanskrit textbooks at the Foundation and Middle School level will be available in Simple Standard Sanskrit (SSS) to teach Sanskrit through Sanskrit (STS) and make its study enjoyable. Phonetics and pronunciation prescriptions in NEP 2020 apply to the Vedas, the oral tradition of the Vedas and Vedic education, as they are founded upon phonetics and pronunciation.

There is no clear distinction made between arts and science, between curricular and extra-curricular activities, between vocational and academic streams, etc. The emphasis in NEP 2020 is on the development of a multi-disciplinary and holistic education among the sciences, social sciences, arts, humanities and sports for a multi-disciplinary world to ensure the unity and integrity of all knowledge. Moral, human and constitutional values like empathy, respect for others, cleanliness, courtesy, democratic spirit, spirit of service, respect for public property, scientific temper, freedom, responsibility, pluralism, equality and justice are emphasized.

The NEP-2020 at point no. 4.23 contains instructions on the pedagogic integration of essential subjects, skills and abilities. Students will be given a large amount of flexible options in choosing their individual curriculum; but in today's fast-changing world, all students must learn certain fundamental core subjects, skills and abilities to be a well-grounded, successful, innovative, adaptable and productive individual in modern society. Students must develop scientific temper and evidence based thinking, creativity and innovation, aesthetics and sense of art, oral and written expression and communication, health and nutrition, physical education, fitness, health and sport, collaboration and teamwork, problem solving and logical thinking, vocational exposure and skills, digital literacy, coding and computational thinking, ethics and moral reasoning, knowledge and practice of human and constitutional values, gender sensitivity, fundamental duties, citizenship skills and values, knowledge of India, environmental awareness etc. Knowledge of these skills include conservation, sanitation and hygiene, current affairs and important issues facing local communities, the states, the country and the world, as well as proficiency in multiple languages. In order to enhance the linguistic skills of children and to preserve these rich languages and their artistic treasures, all students in all schools, public or private, shall have the option of learning at least two years in one classical language of India and its related literature.

The NEP-2020 at point no. 4.27 states that -"Knowledge of India" includes knowledge from ancient India and its contributions to modern India and its successes and challenges, and a clear sense of India's future aspirations with regard to education, health, environment, etc. These elements will be incorporated in an accurate and scientific manner throughout the school curriculum wherever relevant; in particular, Indian Knowledge Systems, including tribal knowledge and indigenous and traditional ways of learning, will be covered and included in mathematics, astronomy, philosophy, yoga, architecture, medicine, agriculture, engineering, linguistics, literature, sports, games, as well as in governance, polity, conservation. It will have informative topics on inspirational personalities of ancient and modern India in the fields of medicinal practices, forest management, traditional (organic) crop cultivation, natural farming, indigenous sports, science and other fields.

The NEP-2020 at point no. 11.1 gives directions to move towards holistic and multidisciplinary education. India emphasizes an ancient tradition of learning in a holistic and multidisciplinary manner, including the knowledge of 64 arts such as singing and painting, scientific fields such as chemistry and mathematics, vocational fields such as carpentry, tailoring; professional work such as medicine and engineering, as well as the soft skills of communication, discussion and negotiation etc. which were also taught at ancient universities such as Takshashila and Nalanda. The idea that all branches of creative human endeavour, including mathematics, science, vocational subjects and soft skills, should be considered 'arts', has a predominantly Indian origin. This concept of 'knowledge of the many arts' or what is often called 'liberal arts' in modern times (i.e., a liberal conception of the arts) will be our part of education system.

At point No. 11.3 the NEP-2020 further reiterates that such an education system "would aim to develop all capacities of human beings -intellectual, aesthetic, social, physical, emotional, and moral in an integrated manner. Such an education will help develop well-rounded individuals that possess critical 21st century capacities in fields across the arts, humanities, languages, sciences, social sciences, and professional, technical, and vocational fields; an ethic of social engagement; soft skills, such as communication, discussion and debate; and rigorous specialization in a chosen field or fields. Such a holistic education shall be, in the long term, the approach of all undergraduate programmes, including those in professional, technical, and vocational disciplines."

The NEP-2020 at point no. 22.1 contains instructions for the promotion of Indian languages, art and culture. India is a rich storehouse of culture – which has evolved over thousands of years, and is reflected in its art, literary works, customs, traditions, linguistic expressions, artifacts, historical and cultural heritage sites, etc. Traveling in India, experiencing Indian hospitality, buying beautiful handicrafts and handmade clothes of India, reading ancient literature of India, practicing yoga and meditation, getting inspired by Indian philosophy, participating in festivals, appreciating India's diverse music and art and watching Indian films are some of the ways through which millions of people around the world participate in, enjoy and benefit from this cultural heritage of India every day.

In NEP-2020 at point no. 22.2 there are instructions about Indian arts. Promotion of Indian art and culture is important for India and to all of us. To inculcate in children a sense of our own identity, belonging and an appreciation of other culture and identity, it is necessary to develop in children key abilities such as cultural awareness and expression. unity, positive cultural identity and self-esteem can be built in children only by developing a sense and knowledge of their cultural history, art, language and tradition. Therefore, the contribution of cultural awareness and expression is important for personal and social well-being.

The core Vedic Education (Vedic Oral Tradition / Veda Path / Veda Knowledge Tradition) of Pratishthan along with other essential modern subjects- Sanskrit, English, Mother tongue, Mathematics, Social Science,

Science, Computer Science, Philosophy, Yoga, Vedic Agriculture, Indian Art, Socially useful productive work etc., based on the IKS inputs are the foundations/sources of texts books of Pratishthan and Maharshi Sandipani Rashtriya Veda Sanskrit Shiksha Board. These inputs are in tune with the NEP 2020. The draft books are made available in pdf form keeping in view the NEP 2020 stipulations, requirements of MSRVVP students and the advice of educational thinkers, authorities and policy of Maharshi Sandipani Rashtriya Veda Vidya Pratishthan, Ujjain. These books will be updated in line with NCFSE in future and finally will be made available in print form.

The Teachers of Veda, Sanskrit and Modern subjects in Rashtriya Adarsh Veda Vidyalaya, Ujjain and many teachers of Sanskrit and modern subjects in aided Veda Pathshalas of Pratishthan have worked for last two years tirelessly to prepare and present Sanskrit and modern subject text books in this form. I thank all of them from the bottom of my heart. Many eminent experts of the national level Institutes have helped in bringing quality in the textbooks by going through the texts from time to time. I thank all those experts and teachers of the schools. I extend my heartfelt gratitude to all my co-workers who have worked for DTP, drawing the sketches, art work and page setting.

All suggestions including constructive criticism are welcome for the improvement of the quality of the text books.

आपरितोषाद् विदुषां न साधु मन्ये प्रयोगविज्ञानम्।
बलवदपि शिक्षितानाम् आत्मन्यप्रत्ययं चेतः॥

(Abhijnanashakuntalam 1.02)

Until the scholars are fully satisfied about the content, presentation, attainment of objective, I do not consider this effort to be successful, because *even the scholars are not fully confident in the presentation without feedback from the stakeholders.*

Prof. ViroopakshaV Jaddipal

Secretary

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Maharshi Sandipani Rashtriya Veda Sanskrit Shiksha Board, Ujjain

FOREWORD

In India, there has been a rich tradition of mathematics. Since ancient times, Indian scholars and mathematicians have excelled in this field. The concept of mathematical knowledge has been given a prominent place since the Vedic period. For example, in the Shankar Bhashya of the Brihadaranyaka, Shankar clearly explains the concept of number lines:

एकप्रभृत्यापरार्धसंख्यास्वरूपपरिज्ञानाय रेखाध्यारोपणं कृत्वा एकेयं रेखा दशेयं, शतेयं, सहस्रेयं इति ग्राहयति, अवगमयति, संख्यास्वरूपम्, केवलं, न तु संख्यायाः रेखातत्त्वमेव

(बृहदारण्यक, शाङ्करभाष्यः 4/4/25)

In other words, a number line can represent a single unit, ten units, a hundred units, a thousand units, and so on. We understand addition, subtraction, and other operations by understanding the number line.

This textbook aims to include mathematical concepts available in Vedic mathematics along with the Sanskrit knowledge system to enhance modern mathematics through the use of ancient knowledge and achievements. The attempt is made to simplify calculations through Vedic mathematics.

In the changing global scenario, the National Education Policy 2020 is designed to provide mathematical proficiency to Vedic students across India by aligning the curriculum and textbooks with the main principles such as discussion, analysis, examples, and applications.

The language of the textbook is very simple and easy to understand, making it accessible for students. This textbook, designed for the fourth (ninth equivalent) year in Vedic schools, aligns with the class 9 mathematics curriculum across India. The book includes various mathematical concepts along with Vedic evidence in the Sanskrit knowledge system. Additionally, references to ancient texts such as Brahmasphuta Siddhanta, Shulba Sutras, Aryabhatiya, Leelavati, and Bijaganita are integrated. This approach allows Vedic students to comprehend not only modern mathematics but also ancient mathematical concepts, experiencing the richness of their Indian heritage. Drawing from previous class experiences, the textbook comprises ten chapters tailored to the needs of the fourth-year curriculum in Vedic schools. Chapter 1 provides a comprehensive overview of numerical systems, exploring both measurable and immeasurable numbers. Chapter 2 discusses polynomials, and Chapter 3 teaches solving linear equations with two variables. Chapter 4 presents mathematical cultures and the verification of answers through Vedic mathematical principles. Chapter 5 delves into the details of circles, and Chapter 6 outlines the Bodhayana (Pythagorean) theorem. Chapter 7 introduces Hero's formula for finding the area of a triangle, and Chapter 8 teaches the surface area and volume of a cube and cuboid using formulas. Chapter 9 explores statistics under the category "Collection of Figures," covering line graphs and bar charts. Chapter 10 focuses on determining the outcomes of possible events through probability. The textbook aims to develop the understanding of mathematical concepts among Vedic students and enhance their ability to rediscover facts. Throughout the book, various **'Do and Learn'** activities are provided for this purpose. At the end of each chapter, important concepts and results are highlighted as **"We Learned"** to reinforce learning.

The book concludes by acknowledging the contributions of Indian mathematicians to the field of mathematics.

The textbook emphasises the importance of understanding this legacy for students preparing for competitive exams. After studying this textbook, students are encouraged to further explore NCERT books for class 9 and subject-specific books.

The author appreciates your constructive suggestions for improving the accuracy of the textbook.

CONTENTS

Chapter 1

Number System

Dear Students, In previous classes you have studied many types of number systems.You must be aware of different types of number systems. Natural numbers such as (1 , 2, 3, 4,..........), Whole numbers (0, 1, 2, 3, 4, …….), Even numbers (2, 4, 6,..........), odd number (1, 3, 5, 7, 9, ………) Integers (…………-3, -2, -1, 0, 1, 2, 3 ……) (fraction and decimals.). Do you know about rational numbers? You will remember that- the number which $\frac{p}{q}$ where p and q are integers, and q ≠ 0. Any rational number is said to be in its standard form when its denominator is a positive integer the numerator and denominator of the rational number have no common factors other than 1.

Example: $\frac{1}{2}, \frac{3}{5}, \frac{-17}{23}, \frac{-12}{1}, \frac{-15}{71}, \frac{-71}{15}$

❖ Discuss with your Guruji, Why do we emphasize that $q \neq 0$ in the general form of rational number $\frac{p}{q}$?

Recall that

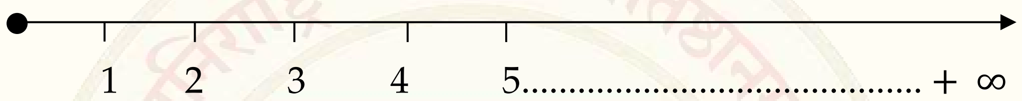
- In the general form of a rational number, dividing the integer p by q gives the quotient as an integer or a decimal number.
- Rational numbers include integers, fractions and decimals (both positive and negative).

- infinite rational numbers are there between any two rational numbers.

You must have studied about the numbers discussed above, in the previous classes. In this chapter we will study rational and irrational numbers in detail.

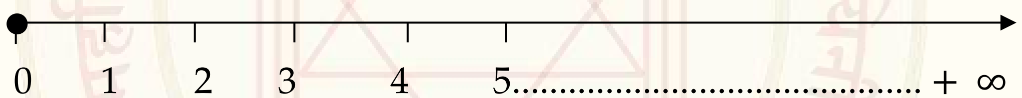
Revision of Number System on the Number line -

1) Natural numbers –



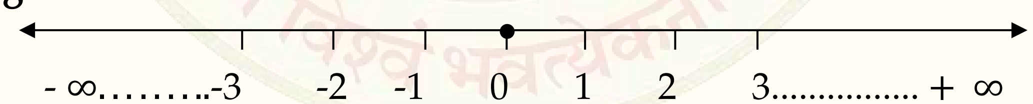
In the above number line, the numbers start from 1 and increases towards right direction.

2) Whole number –



In the above number line, the numbers starts from 0 and increases in the right direction.

3) Integers –



In the above number line, to the right of 0 the numbers increase and to its left the numbers decrease infinitely.

❖ Do and Learn:

Choose the correct answer:

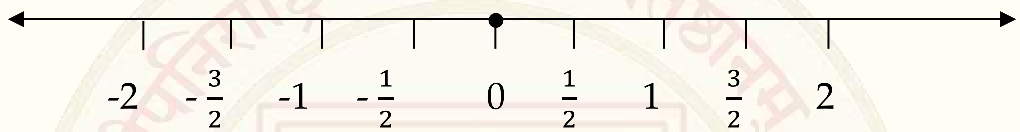
(a) Every whole number is a natural number –

(I) True (II) False (III) Not possible (IV) None of these

(b) Every natural number is a whole number –

(I) True (II) False (III) Not possible (IV) None of these

Rational Number -



The numbers which can be written in the form of a fraction numerator and denominator or $\frac{p}{q}$ where p, q are integers and $q \neq 0$, are called Rational numbers.**There are infinite rational numbers between any two rational numbers.** Further in this sequence we will study about irrational numbers.

Irrational Number -

The numbers which are not rational numbers are called **irrational numbers**, They Cannot be written in the form of $\frac{p}{q}$. You know that there are infinite rational numbers between two numbers, similarly there are infinite irrational numbers between two numbers.

Example: $\sqrt{2}$,$\sqrt{3}$, $\sqrt{5}$,$\sqrt{7}$,π, 0.16160016000160000..............

The numbers given in above example can not be expressed in the form of $\frac{p}{q}$.

Note: Rational number includes recurring decimals.

Irrational numbers include -

- Square root of non-perfect squares.
- Cube root of non-perfect cubes.
- None terminating and non-recurring numbers.

You know that there are infinite rational numbers between any two numbers, similarly there are infinite irrational numbers between any two numbers.

For example: between 0.101001000… and 0.110110011000... there are infinite numbers.

You will remember that whenever we use the symbol "$\sqrt{\ }$" Square root, we consider that the number is a positive square root, for example $\sqrt{16} = 4$, even though 4 and (- 4) both are square roots of 16.

Hence, the group of rational and irrational numbers taken together on the number line is called **real numbers,** real numbers are denoted by 'R', are there real numbers in the group of all types of numbers? Yes, the group of rational and irrational numbers is called real numbers.

Properties of Rational and Irrational Numbers

Properties of Rational Numbers

1. Rational numbers can be represented on the number line just like integers.

2. the result obtained by adding, subtracting, multiplying or dividing(If defined) two rational numbers, unique rational number is obtained. For example,

   sum - $\frac{2}{5}$, $\frac{4}{5}$, $\frac{6}{5}$ (It is a rational number.)

   subtraction - $\frac{4}{5} - \frac{3}{5} = \frac{1}{5}$ (It is a rational number.)

3. The product of any two rational numbers is a rational number. Similarly the result of division of two rational is a rational number.**Example:**

   multiplication- $\frac{3}{6} \times \frac{2}{5} = \frac{6}{30}$ or $\frac{1}{5}$ (It is a rational number.)

   Division- $\frac{2}{5} \div \frac{3}{6} = \frac{2}{5} \times \frac{6}{3} = \frac{12}{15}$ or $\frac{4}{5}$ (It is a rational number.)

4. The operations of addition and multiplication of rational numbers are commutative associative.

   sum- $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = \frac{4}{5} + \frac{2}{5}$

   Multiplication- $\frac{2}{5} \times \frac{3}{6} = \frac{6}{30} = \frac{3}{6} \times \frac{2}{5}$

Properties of Irrational Numbers :

1. When two numbers, one of which is rational and the other is irrational, are added, we get an irrational number. **Example:**

   (3) + (4 $\sqrt{5}$)= 3 + 4 $\sqrt{5}$ (It is an Irrational number.)

   5 $\sqrt{5}$+ 7 $\sqrt{5}$= 12 $\sqrt{5}$ (It is an Irrational number.)

2. If a rational number and an irrational number are multiplied, we will get an irrational number only. **Example:**

   5 × 2 $\sqrt{5}$= 10 $\sqrt{5}$ (It is an Irrational number.)

$3\sqrt{5} \times 4 = 12\sqrt{5}$ (It is an Irrational number.)

3. The product of two irrational numbers can be rational or irrational numbers. **Example:**

   $\sqrt{7} \times 2\sqrt{7} = 2 \times 7 = 14$ (It is a rational Number.)

   $3\sqrt{2} \times 4\sqrt{2} = 12 \times 2 = 24$ (It is a rational number.)

Rational and irrational numbers can be differentiated by observing the digits in their decimal parts.

decimal expansion of rational Numbers –

A number whose decimal part is terminating or non-terminating repeating. (Recurring) is called a rational number.

a) Rational numbers whose decimal expansion is terminating -

Example : $\frac{1}{2}$, $\frac{8}{10}$, $\frac{5}{8}$

$\frac{1}{2} = 0.5$ (terminating decimal)

$\frac{8}{10} = 0.8$ (terminating decimal)

$\frac{5}{8} = 0.625$ (terminating decimal)

B) rational numbers whose decimal expansion is non-terminating recurring

In a rational number $\frac{p}{q}$, on dividing p by q continuously, if the reminder repeats at regular intervals, correspondingly the digits in the quotient also repeat at the same intervals. Such numbers are also rational numbers.

Example:

$\frac{1}{3}$ = 0. 3333............ Recurring decimal

$\frac{1}{7}$ = 0.142857142857…. Recurring Decimal

In $\frac{1}{3}$ The repetition of 3 in the quotient of $\frac{1}{3}$ written as $0.\overline{3}$ or 0.333..., and similarly in $\frac{1}{7}$ the quotient of $\frac{1}{7}$ in which the repetition of digits 142857 is written as $0.\overline{142857}$.

Hence, we can conclude that **terminating and recurring decimals** are **rational number.**

❖ **Do and Learn –**

Select the terminating and non-terminating rational numbers from the following decimal numbers.

1) 0.254 2) 0.333 3) $0.\overline{483}$

4) $1.\overline{25}$ 5) $2.\overline{45}$ 6) 4.825

Decimal expansion of irrational numbers –

The decimal numbers which are non-terminating and non-repeating (non-recurring) are called irrational numbers. In other words, decimal representation of irrational numbers are non-terminating and non-recurring.

Reference about the method of finding the square root of a number is found in our Shulbha Sutras. Reference about $\sqrt{2}$ is given in the following verse taken from Baudhayan, shulbasutra 2. 12.

प्रमाणं तृतीयेन वर्धयेत्तच्च चतुर्थेनात्मचतुस्त्रिंशोऽनेन सविशेषः ॥

(बोधायन शुल्बसूत्र 2.12)

Meaning, to the side of a unit square, add the result of the sum of $(\frac{1}{3})^{rd}$of the side of the square, and $(\frac{1}{4})^{th}$ of $(\frac{1}{3})^{rd}$ of the side of the square, and subtract $(\frac{1}{34})^{th}$of the $(\frac{1}{4})^{th}$ of the$(\frac{1}{3})^{rd}$of the side of the square, to get the value of square root of 2.

this formula Explains the value of $\sqrt{2}$

$$\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3 \times 4} - \frac{1}{3 \times 4 \times 34} = 1.414256$$

the above $\sqrt{2}$ is 1.414256. Presently in refined form the value of decimal representation is 1.4142135623…

Since The decimal form of $\sqrt{2}$ is non-terminating and non-recurring, $\sqrt{2}$ is an irrational number. Do you know the value of π? π is an irrational number because its decimal form is non – terminating and non – repeating (recurring).

The approximate value of $\pi = \frac{22}{7}$

The decimal form of π = 3.14592653………

Definition of irrational number -

the number which can not be expressed in the form of $\frac{p}{q}$ where p and q are integers or **decimal** number which are **non**- terminating and non-recurring are called **an irrational number**.

Example:

0.2354...... ,0.0808008000...... , π, 0.16160016000160000......

Do and Learn –

Match the decimal number with terminating, non – terminating and recurring, and non- terminating and non-recurring numbers.

terminating - 3.141414

non-terminating - 1.25

non- terminating recurring - 3.14592653........

Exercise 1.1

1. Select the correct option for the following multiple choice questions.

(a) Which of the following is a rational number?

(I) 0 (II) $\sqrt{2}$ (III) π (IV) None of these

(b) Which of the following is a rational number?

(I) $\sqrt{2}$ (II) $\sqrt{23}$

(III) $\sqrt{225}$ (IV) 0.1010010001………

(c) Which of the following is a rational number?

(I) $\frac{3}{\sqrt{2}}$ (II) $\frac{1}{2}$ (III) $\sqrt{2}$ (IV) $\sqrt{11}$

(d) Which of the following is a rational number?

(I) $\sqrt{2}$ (II) $\sqrt{3}$ (III) $\sqrt{4}$ (IV) $\sqrt{5}$

(e) Which of the following is an irrational number?

(I) 0 (II) 1 (III) 2 (IV) $\sqrt{2}$

(f) π is a/an -.

(I) Rational numbers (II) Irrational numbers

(III) Whole number (IV) none of these

(g) zero is a/an-.

(I) Rational numbers (II) Irrational numbers

(III) Natural numbers (IV) none of these

(h) Every rational number is –

(I) A natural number (II) Awhole number

(III) An integer (IV) A real number

(i) Every irrational number is a real number -

(II) True (II) False (III) Not possible (IV) None of these

(j) Every rational and irrational numbers comes under the group of …………..

(III) Natural numbers (II) Whole numbers

(III) Integers Number (IV) Real Number

2. Select the terminating, non- terminating and non- recurring, non-terminating and recurring numbers from the following decimal numbers.

a) 0.36 b) 0.0909090909 Or $0.\overline{9}$

c) 4.125 d) 0.230739230......

e) 0.1818 f) 0.8225

3. Sort out rational numbers and irrational numbers from the following decimal numbers.

a) 0.36 b) 0.3796 c) 0.8225

d) $0.\overline{142857}$ e) 1.10100100010000

f) 1.25 g) 7.478478

4. Sort out rational and irrational numbers from the following numbers.

a) $\sqrt{2}$ b) $\sqrt{16}$ c)$\sqrt{23}$ d) $\sqrt{25}$

e) $\sqrt{36}$ f) $\sqrt{35}$ g) $\sqrt{49}$ h) $\sqrt{50}$

Operations on real numbers –

The addition and subtraction of irrational numbers is mentioned in beejaganitam written by Bhaskaracharya which is given below.

योगं करण्योर्महतीं प्रकल्प्य वधस्य मूलं द्विगुणं लघुं च ।

योगान्तरे रूपवदेतयो: स्तो वर्गेण वर्गं गुणयेद्भजेच्च ॥

(बीजगणितम् ,अथ करणीषड्विधम् ,13)

Meaning, addition and subtraction of numbers with the same irrational parts are done. In other words, in addition and subtraction of numbers having the same irrational part, the rational parts are added or subtracted, as explained in **'Dhanarnayorantarmev Yoga:'** Also, the number whose complete root is not found is kept in its original form (Mula Karani), the result is written with the same irrational part and sum and difference of the rational part of the given numbers.

Addition and Subtraction

Example : Find the sum of $2\sqrt{2}$and $\sqrt{2}$.

Solution : $2\sqrt{2}+\sqrt{2}=3\sqrt{2}$

Example : Find the sum of $3\sqrt{5}+3\sqrt{2}$ And $2\sqrt{5}$

Solution: $3\sqrt{5}+3\sqrt{2}+2\sqrt{5}$

$= 3\sqrt{5}+2\sqrt{5}+3\sqrt{2}$

$= (3\sqrt{5}+2\sqrt{5})+3\sqrt{2}$

$= 5\sqrt{5}+3\sqrt{2}$

Example : $3\sqrt{5}$ Subtract from$7\sqrt{5}$.

Solution : $7\sqrt{5} - 3\sqrt{5} = 4\sqrt{5}$

Example : Subtract $2\sqrt{3}$ out of $9\sqrt{3}+5\sqrt{2}$.

Solution : $(9\sqrt{3}+5\sqrt{2})-(2\sqrt{3})$

$= (9\sqrt{3}-2\sqrt{3})+5\sqrt{2}$

$= (9\sqrt{3}-2\sqrt{3})+5\sqrt{2}$

$= 7\sqrt{3}+5\sqrt{2}$

Multiplication–

We will learn to perform the operation of multiplication between irrational numbers using the following examples.

1) $\sqrt{2}\times\sqrt{2} = 2$ (since $2=\sqrt{2}.\sqrt{2}$)

2) $3\sqrt{2}\times 3\sqrt{2} = 9(\sqrt{2}\times\sqrt{2}) = 9\times 2 = 18$

3) $\sqrt{2}\times\sqrt{3} = \sqrt{2\times 3} = \sqrt{6}$

4) $3\sqrt{2}\times 3\sqrt{3} = 9\sqrt{2\times 3} = 9\sqrt{6}$

Division –

We will learn the operation of division of irrational numbers using the following examples.

Example : Divide $8\sqrt{2}$ by $3\sqrt{2}$.

Solution : $8\sqrt{2} \div 3\sqrt{2}$

$= \frac{8\sqrt{2}}{3\sqrt{2}}$

$= \frac{8}{3}$

Example : Divide $8\sqrt{15}$ by $3\sqrt{5}$.

Solution: $8\sqrt{15} \div 3\sqrt{5}$

$= \frac{8\sqrt{15}}{3\sqrt{5}}$

$= \frac{8\sqrt{5}\sqrt{3}}{3\sqrt{5}}$

$= \frac{8\sqrt{3}}{3}$

The meaning of simplification of an irrational number is to express it as the sum of a rational and an irrational number.

Consider the number $\frac{1}{\sqrt{2}}$.

$\frac{1}{\sqrt{2}}$ is an irrational number. If the denominator of an expression contains a square root term or an irrational term, the method of converting the denominator of the equivalent expression into a rational form is called rationalization of the denominator.

The method of rationalization has been explained in beejaganitam written by Acharya Bhaskaracharya.

धनर्णताव्यत्ययमीप्सितायाश्छेदे करण्या असकृद्विधाय ।

तादृक् छिदा भाज्यहरौ निहन्या- देकैव यावत्करणी हरे स्यात् ॥

(करणीषड्विधम् 16 पृ. 63)

Meaning: To rationalise the denominator of an irrational number, multiply and divide by its rationalising factor, which is obtained by changing the sign of the irrational term in it.

Example: Rationalize the denominator of the number. $\frac{1}{\sqrt{2}}$

Solution: To write $\frac{1}{\sqrt{2}}$ to with a rational number in the denominator, we know that, $2 = \sqrt{2} \times \sqrt{2}$ is a rational number, hence, multiplying $\frac{1}{\sqrt{2}}$ and $\frac{\sqrt{2}}{\sqrt{2}}$ we get an equivalent expression, because $\frac{\sqrt{2}}{\sqrt{2}} = 1$ thus, considering these two facts together we get.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

In this form, determining the location of the number $\frac{1}{\sqrt{2}}$ On a number line becomes easy. It is located in the middle of 0 and $\sqrt{2}$.

Example: Rationalize the denominator of $\frac{1}{2+\sqrt{3}}$.

Solution: $\frac{1}{2+\sqrt{3}}$ (Multiplying and dividing by 2 - $\sqrt{3}$) we get.

$$\frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2}$$

$$= \frac{2-\sqrt{3}}{4-3}$$

$$= \frac{2-\sqrt{3}}{1}$$

$$= 2-\sqrt{3}$$

We know that-

(a+b)(a-b) $= a^2$- b^2

Example : Rationalize the denominator of : $\frac{3}{\sqrt{5}-\sqrt{3}}$

Solution: $\frac{3}{\sqrt{5}-\sqrt{3}}$ On multiplying and dividing by $(\sqrt{5}+\sqrt{3})$

$$\frac{3}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{3(\sqrt{5}+\sqrt{3})}{5-3}$$

$$= \frac{3(\sqrt{5}+\sqrt{3})}{2}$$

Exercise 1.2

1. Select the correct option for the following multiple choice questions.

   (a) $6\sqrt{5} \times 6\sqrt{5} =$

   (I) $12\sqrt{5}$ (II) 60 (III) 180 (IV) $8\sqrt{5}$

   (b) $8\sqrt{15} \div 2\sqrt{3} =$

   (I) $16\sqrt{45}$ (II) $16\sqrt{5}$ (III) $4\sqrt{5}$ (IV) 720

2. Which of the following numbers are rational numbers?

   1) $\frac{2\sqrt{7}}{5\sqrt{7}}$ 2) $\frac{1}{\sqrt{2}}$ 3) $3\sqrt{3} - \sqrt{3}$ 4) 3π

3. Simplify.

   1) $3\sqrt{5} + 2\sqrt{5}$ 2) $4\sqrt{2} + 3\sqrt{2}$ 3) $5\sqrt{3} + 8\sqrt{3}$

   4) $8\sqrt{3} - 5\sqrt{3}$ 5) $7\sqrt{5} - 3\sqrt{5}$ 6) $\sqrt{3} \times \sqrt{3}$

   7) $4\sqrt{3} \times 3\sqrt{3}$ 8) $\frac{8\sqrt{15}}{\sqrt{5}}$ 9) $\frac{5\sqrt{16}}{\sqrt{8}}$

4. Rationalize the denominator of the following.

   1) $\frac{1}{\sqrt{7}}$ 2) $\frac{1}{\sqrt{7} - \sqrt{6}}$ 3) $\frac{1}{\sqrt{5} + \sqrt{2}}$ 4) $\frac{1}{\sqrt{4} + \sqrt{3}}$

   5) $\frac{8}{\sqrt{10}}$ 6) $\frac{5}{\sqrt{8} - \sqrt{6}}$ 7) $\frac{5}{\sqrt{12} + \sqrt{2}}$ 8) $\frac{3}{\sqrt{4} + \sqrt{3}}$

We learned -

1) Rational numbers are those numbers which can be written as $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called rational numbers.

   Example: - $\frac{2}{3}$ can be written as $\frac{p}{q}$, where p = 2, q = 3 both are integers and q is not equal to zero. Therefore $\frac{2}{3}$ is a rational number. Other example of rational numbers are $\frac{2}{3}$, $\frac{12}{33}$, $\frac{-2}{3}$ etc.

   Rational numbers are represented by 'Q'.

2) Irrational Numbers are the numbers which are not rational. Which means the number which cannot be expressed in the form of a fraction p/q, where p and q are integers, $q \neq 0$ are called irrational numbers. Irrational numbers are of the form ($\sqrt{\ }$)

   For example:

   $\sqrt{2}$, $\sqrt{5}$, $\sqrt{13}$,$\sqrt{33}$ ……….

3) The decimal form of a rational number is either terminating or non-terminating recurring.

4) The decimal form of rational number can either be terminating or non – terminating and recurring.

5) Rational and irrational numbers together gives us the set of real numbers. The set of real numbers is denoted by 'R'.

6) To rationalize the denominator of $\frac{1}{\sqrt{a}+\sqrt{b}}$, we multiply it with $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}$ Where a and b are positive integers.

Chapter 2

Polynomials

Dear students ! You have learned about operations (+, − , ×, ÷) on algebraic expressions in your previous classes. We can also factorise algebraic expressions.

Example: $y^2 = y \times y$

$x^3 y = x \times x \times x \times y$

You will remember about algebraic identities and their factors.

Example:

$$(x + y)^2 = x^2 + 2xy + y^2$$
$$(x - y)^2 = x^2 - 2xy + y^2$$
$$x^2 - y^2 = (x + y)(x - y)$$

In this chapter, we will first study about the remainder theorem along with the terminologies related to polynomials.

polynomial -

In the following verse of, beejaganitam we find the following verse about representing constants and variable quantities.

यावत्तावत्कालको नीलकोऽन्यो वर्णः पीतो लोहितश्चैतदाद्याः ।

अव्यक्तानां कल्पिता मानसंज्ञा-स्तत्संख्यानं कर्तुमाचार्यवर्यैः ॥

To calculate the unknowns or variable quantities, there are mentions like yavat, Tavat Kalak, Nilak, Pitak and Lohit etc. are used, so that all the variable quantities can be known separately. A variables is represented by x, y, z……. Similarly, an algebraic expression can be formed using operations (+, – , ×, ÷) on constants and variables.

Generally in the form of algebraic expression **ax.** In which **a** is a **constant** and **x** is **a variable.**

Example : $4x, 9x, x^2 + 2, x + 5, x^3 + 4x$………… Etcetera.

Example, these are algebraic expressions, in which the exponents are whole number (0, 1, 2, 3 …..). The polynomial is denoted by P(x). In other words, it can be said that "The algebraic expression in which the degree of the variables is a whole number (0 , 1, 2, 3 ……..) is called a **polynomial".**

Let us understand polynomials using examples.

Example : The algebraic expression $x^2 + 2x + 1$ is a polynomial.

Answer : Yes , this algebraic expression is a polynomial because the variable in this polynomial is x, the exponent of the variable x are whole number (2 and 1).

Example: Is the algebraic expression $x^2 + 2y^5 + 1$, a polynomial ?

Answer : In the given algebraic expression the variables are x and y. The exponent of both the variables are (2 and 5) whole number.

Hence $x^2 + 2y^5 + 1$ is a polynomial.

Diksha: Is $3 + t^6$ also a polynomial?

Vishal: Yes, this is also a polynomial because in this the exponent of the variable t is 6 which is a whole number.

Diksha: Is $(3x^3 + 4y^{-5} + 3)$ also a polynomial? How many variables are there in this algebraic expression?

Vishal: This algebraic expression has two variables x and y. It is not a polynomial because the exponent of the variable y is (-5) which is not a whole number.

Guru ji: You are discussing absolutely right. **The algebraic expression, in which the exponent of the variable is a whole number is called a polynomial.** Such as $x^2 + 2$, $x^2 + y^2 + t^3$, $3x + 5$

On the other hand, the algebraic expression in which -

1) The exponent of the variable is negative. $-2, -5, -3$

2) Any term which is divided by any variable like $\frac{1}{x}$, $\frac{1}{3x}$, $3\,x + \frac{5}{x}$

3) Any different exponent like $\sqrt{x}$ because it is written like $x^{\frac{1}{2}}$.

For Example: In $x^2 + y^{\frac{1}{2}}$, Exponent of in variable y, in the algebraic expression is $\frac{1}{2}$ (fraction) is therefore $x^2 + y^{\frac{1}{2}}$ is not a polynomial.

If **the exponent of the variable in an algebraic expression is negative or a fraction or any term that is divisible by a variable, the algebraic expression is not a polynomial.** But a polynomial can have constants , variables with degrees of whole numbers.

Example :

Constants = $3, 2, -5, \frac{1}{2}$

Variables = x , xy , xyz, abc

Exponents = 0, 1, 2, 3 etc.

Degree of Polynomial –

If there is a polynomial P(x), then the highest power of any variable in that polynomial is called the degree of **the polynomial.** Degree of Polynomial.

1) The polynomial in which the highest degree of the variable is 1, is called a linear polynomial or a polynomial of degree 1.

   For example : $3x$, $4x + 1$, $1 + 5x$

2) The polynomial in which the highest degree of the variable is 2 is called **quadratic polynomial.**

   For example : $x^2 + x + 2$, $2x^2 + 1$, $3 + 3t^2$

3) The polynomial in which the highest degree of the variable is three is called **cubic polynomial.**

   For example : $3x^3 + 3x$, $3x^3 + x^2 + 2x + 1$, $4x^3$

4) Constant polynomial is called **zero polynomial**. In other words, the maximum degree of zero polynomial is zero.

   For example : $3x^0 + 1$, 4 , $-5y^0$

   (Since $x^0 = 1$)

Think : 1) Is $4\ x^0 + 1$ also a polynomial? (yes/no)

2) Is $3\sqrt{x} + 2$ also a polynomial? (yes/no)

Do and Learn –

Write two examples for the following.

linear polynomial, quadratic polynomial and cubic polynomial

a) Linear polynomial $\underline{3x+5}$, ________ , ________

b) Quadratic polynomial $\underline{t^2+2t}$, ________ ,________

c) Cubic polynomial $\underline{3t^3+2t^2+1}$, ________ , ________

Example : Whether the algebraic expressions given below are polynomials or not? If yes, write the number of terms of the polynomial and the degree of the polynomial.

1). $3x^4 + 2x^2 + 1$ **2).** $3\ x^2 + 2\sqrt{x} + 1$

Solution 1) : $3x^4 + 2x^2 + 1$

Given – $3x^4 + 2x^2 + 1$ The algebraic expression is a polynomial.

Number of terms in polynomial = 3 ($3x^4$, $2x^2$, 1)

The degree of the polynomial is 4.

Solution 2): $3x^2 + 2\sqrt{x} + 1$

Given: $3x^2 + 2\sqrt{x} + 1$ is not a polynomial because the coefficient of 2 is $\sqrt{x}$ That is, the degree of $x^{½}$ variable is a fraction (1/2), hence it is not a polynomial.

Exercise 2.1

1. Select the correct option for the following multiple choice questions.

(a) Which of the following is a linear polynomial?

(I) x^2 (II) $7x^3$ (III) x-x^3 (IV) x+1

(b) Which of the following is a quadratic polynomial?

(I) x - x^3 (II) 1 + x (III) y+ y^2+ 1 (IV) $3t$

(c) Which of the following is a quadratic polynomial?

(I) x^3+ x (II) $x^2 + x$ (III) x+1 (IV) x^3

(d) Which of the following is a cubic polynomial?

(I) x^2 (II) x +1 (III) $7x^3$ (IV) $\frac{1}{x^3}$

(e) The degree of polynomial 5 x^3+ 4 x^2+ $7x$ will be -

(I) 1 (II) 2 (III) 3 (IV) 4

(f) polynomial 5 y^6 $-4x^2$ The power of -6x is -

(I) 2 (II) 3 (III) 6 (IV) 8

2. Which of the following expressions is a polynomial ?

(1) $3x^2 + 1$ (2) $3x^2 + 4t^2 + 1$ (3) $2x^{-5} + 2x^2 + 1$

(4) $3\sqrt{x}+1$ (5) $10x^5 + 3x$ (6) $4x^{-7} + 3x - 5$

3. Sort out linear polynomials, quadratic polynomials and cubic polynomials from the following polynomials.

($3x^2 + 4x + 1$, $4x^2$, $3x + 1$, $4x^2 + t + 1$,

$5x^2$, $x^3 + 1$, $4y + 1$, $5y$, $2x^2+1$,$x + 3t^3$)

4. What is the degree of the following polynomials ?

(1) $10x^{10} + 10$ (2) $5x^7 + 5x^6 + 1$ (3) $3x^2 + 2$

(4) $5x^2$ (5) $10y^3 + y + 1$ (6) $10x^9$

(7) $10x^0 + 1$ (8) 14 (9) $10x^7 + 10x^0 + 1$

(10) $5x^1 + 5x^0$ (11) 0 (12) $6x^1 + 7x^8$

Zeros of polynomial -

Consider the following polynomial p(x).

$$p(x) = 3x + 1$$

If 2 is substituted in place of the variable in the polynomial $p(x)$, then the value of this polynomial $p(x)$ will be obtained as follows.

$$p(2) = 3(2) + 1$$

$$= 6 + 1 \qquad [\,3(2) = 3 \times 2 = 6\,]$$

$$= 7$$

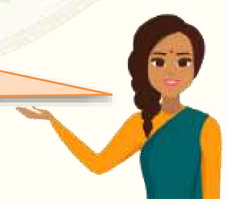
We can say that $x = 2$ But The value of the polynomial $p(x)$ is 7.

Thus , $p(0) = 3 \times 0 + 1 = 0 + 1 = 1$

$(3) = 3 \times 3 + 1 = 9 + 1 = 10$

Laxmi -

Can you find p(4) ?



In a polynomial, if we substitute a value in place of the variable in which the value of the polynomial is zero, the value of the variable is called **zero of the polynomial.** A real number k is called the zero of the polynomial p (x) if P (k) = 0.

Let us understand the zeros of a polynomial using examples.

Example : Check whether 2 or (-2) are the zeros of the polynomial

$p(x) = x + 2$?

Solution: $p(x) = x + 2$

When $x = 2$ on substituting $p(2) = 2 + 2 = 4$

When $x = -2$ on substituting $p(-2) = -2 + 2 = 0$

Hence (–2) is the zero of the polynomial $p(x) = x + 2$ and 2 is not a zero of the polynomial.

Example: Check whether 3 or – 3 the polynomial is the zero of the polynomial $p(y) = 2y - 6$?

Solution : $p(y) = 2y - 6$

Substituting y = 3, $p(3) = 2(3) - 6 = 6 - 6 = 0$

Substituting y = (– 3), $p(-3) = 2(-3) - 6 = -6 - 6 = -12$

Hence, 3 is a zero of the polynomial $p(y) = 2y - 6$ whereas (– 3) is not.

Do and Learn:

Marked (√) the correct answer -

The polynomial (x + 10) has a zero. (10 / – 10)

Coefficient of term in Polynomial:

Example: In the Polynomial $\frac{\pi}{2}x^3 + x$, coefficient of x^3 is:

Solution: Given polynomial $\frac{\pi}{2}x^3 + x$

constant factor of $\frac{\pi}{2}x^3$ in the polynomial is $\frac{\pi}{2}$.

Hence $\frac{\pi}{2}$ is the coefficient of x^3.

Example: If x = 5 The value of polynomial $4x^2+3$ will be –

Solution: Given polynomial $4x^2+3$

if $x = 5$ then, $P(x) = 4x^2+3$

$P(5) = 4(5)^2 + 3$

$P(5) = 4\times 25 + 3$

$P(5) = 100 + 3 = 103$

Hence at $x = 5$, the value of polynomial $4x^2+3$ will be 103.

Remainder Theorem –

Let's take two numbers 16 and 3. You know that dividing 16 by 3 will give 5 quotient and remainder is 1. we know ,

Dividend = divisor ×quotient + remainder

$16 = 3 \times 5 + 1$

$16 = 15 + 1$

$16 = 16$

3) 16 (5

– 15

01 (remainder)

We observe here that there is a remainder of

1. Similarly, dividing 10 by 2 we get

10	=	5 x 2 + 0	Divisor	Dividend	Quotient
10	=	10	2)	10	(5
				–10	
				______	
				00	Reminder

In this the remainder is 0.

From this we can say that 2 is a factor of 10 or 10 is a multiple of 2.

Aaradhya – Is it possible to divide one polynomial by another?

In algebra remainder theorem of Polynomial is an application of polynomial division. To find the quotient of algebraic terms, the following verse is found in beejaganitam.

भाज्याच्छेदः शुद्धयति प्रच्युतः सन् स्वेषु स्वेषु स्थानकेषु क्रमेण ।

यैर्यैर्वर्णैः संगुणो यैश्च रूपैर्भागाहारे लब्धयस्ताः स्युरत्र ॥

(बीजगणितम् , भागहारे करणसूत्रं वृत्तम् , पृं. 38)

According to the above mentioned verse of beejaganitam, the method of division of polynomials is clear the of the divisor is multiplied with a quantity such that terms in the dividend exactly divides.

Those alphabets and forms by which the divisors of the multiplication and subtraction from the dividend at their respective places become pure Which means do not have any remainder, those alphabets and forms are here called Labdhi Which means Bhajanphal, that amount or form is obtained in the form of quotient..

Let us understand with an example.

Example : Divide the polynomial $(3x^2 + 4x + x)$ by x.

Solution : $(3x^2 + 4x + 3) \div x$

$$= \frac{3x^2 + 4x + 3}{x}$$

$$= \frac{3x^2}{x} + \frac{4x}{x} + \frac{3}{x}$$

$$= 3x + 4 + \frac{3}{x}$$

Do and Learn -

Divide - $(2y^3 + 2y^2 + y) \div y$

Example: If there are two polynomials p(x) and g(x), then divide p(x) by g(x).

$p(x) = 5x^2 + 7x + 3$, $g(x) = x + 1$

Solution : $p(x) \div g(x)$

$= (5x^2 + 7x + 3) \div (x + 1)$

$$
\begin{array}{r|l}
 & \quad 5x + 2 \\
\hline
x + 1 & 5x^2 + 7x + 3 \\
 & 5x^2 + 5x \\
 & (-) \quad (-) \\
\cline{2-2}
 & \qquad 2x + 3 \\
 & \qquad 2x + 2 \\
 & \qquad (-) \quad (-) \\
\cline{2-2}
 & \qquad\quad 1
\end{array}
$$

quotient : 5x + 2

Remainder : 1

The above given part is completed in the following steps.

Step I: Dividing the first term of the dividend by the first term of the divisor Which means dividing $5x^2$ by x gives 5x.

Step II : Multiplying the divisor by the first term of the quotient 5x , the product obtained $5x^2 + 5x$ is subtracted from the dividend. In this way 2x + 3 is obtained.

Step III : Taking the remainder 2x + 3 as the new dividend and again doing **Step (I)** The process of quotient is adopted, thus the second term of the quotient is obtained as 2.

Step IV : As in **Step (II)**, by multiplying the second term of the quotient with the divisor (x + 1), the product 2x + 2 is subtracted from the dividend 2x + 3, resulting in the remainder 1.

We repeat this process until the degree of the dividend becomes less than the degree of the divisor. In the last step, the dividend becomes the remainder and the sum of the quotients becomes the complete quotient.

The denominator of this example is a linear polynomial, in this we consider the relationship between the remainder and the dividend.

In P (x) = $5x^2 + 7x + 3$

By substituting (–1) in place of x

Since $(-1)^2$

$= -1 \times -1 = 1$

$$P(-1) = 5(-1)^2 + 7(-1) + 3$$
$$= 5 \times 1 + (-7) + 3$$
$$= 5 - 7 + 3$$
$$= 1$$

Therefore, the remainder obtained by dividing the polynomial $P(x) = 5x^2 + 7x + 3$ by $(x + 1)$ is the is obtained by finding the value of the polynomial at the zero of the divisor $(x + 1)$ namely p(–1).

Example : Find the remainder after dividing $(3x^2 + 2x + 1)$ to $(x + 2)$ by.

Solution : $P(x) = 3x^2 + 2x + 1$

Zero of $(x + 2)$ is (-2)

$$P(-2) = 3(-2)^2 + 2 \times (-2) + 1$$
$$= 3 \times 4 + (-4) + 1$$

Since

$x + 2 = 0$

$x = -2$

$= 12 - 4 + 1$

$= 13 - 4$

$= 9$

Hence the remainder is 9.

Do and Learn

1) Find the remainder when polynomial $(4x^2 + 2x + 1)$ divided by $(x + 1)$.

Exercise 2.2

1. Select the correct option from the following multiple-choice questions.

(a) Coefficient of x^2 in the Polynomial $2- x^2 + x^3$ is-

(I) -1 (II) 1 (III) 2 (IV) 3

(b) The coefficient of x^2 in the polynomial $\frac{\pi}{2}x^2 + x$ is-

(I) π (II) $\frac{\pi}{2}$ (III) 2π (IV) $\frac{1}{2}$

(c) The coefficient of x in $x^2 - x + 7$ is-

(I) 1 (II) -1 (III) 2 (IV) 7

(d) The value of polynomial $5x - 4x^2 + 3$ for x = - 1 will be -

(I) 0 (II) 1 (III) -5 (IV) -6

(e) For x = 2 the value of the polynomial $5x - 4x^2 + 3$ will be -

(I) -2 (II) -3 (III) -5 (IV) -6

(f) The value of the polynomial $p(y) = y^2 - y + 1$ at y =0 will be –

(i) 0 (II) 1 (III) 2 (IV) 3

(g) At $x = 0$ the value of $p(x) = x^2$ will be –

(I) 0 (II) 1 (III) 2 (IV) 3

(h) The zero $p(x) = x + 5$ is -

(I) 5 (II) -5 (III) 1 (IV) -1

2. Find the values of polynomials at the following points

$(3x^2 + 2x + 1)$.

a) x = 2 b) x = 1 c) x = 0

3. Check whether 3 or (– 3)are zeros of the Polynomials
$P(x) = x^2 + 9$.

4. Check whether 4 or (– 4) are zeros of the Polynomials
$P(t) = t + 4$.

5. Find the remainder when the polynomial $(2x^2 + 3x + 1)$ is divided by the following.
A) (x + 1) B) (x + 2) C) (x – 1) D) (x – 2)

6. Find the remainder on dividing the polynomial $(5x^4 + 4x^2 + 3x + 1)$ by $(x + 1)$.

We learned -

1) When a variable (x, y, z, ………) and a constant are expressed with four basic operations (+, –, ×, ÷) it is called **an algebraic expression.**

   Example : (1)$x^2 + 2$ (2) $x^{-5} + 4x + 3$

2) The algebraic expression whose exponent of variables is a whole number is called **a polynomial.**

   Example:: $3x^2 + 4$, $4x^{10}$, $5x^2 + 3$

3) On the other hand, the algebraic expression whose degree is not a whole number or a fraction is not called a polynomial.

   Example:

   $3\sqrt{x} + 1$, $3x^{½} + 4$, $x^{-5} + y$, $2x^{-4} + 3$

4) Polynomials having one term are called **monomials.** $5x$, $3x$

5) Polynomials having two terms are called **binomials.**
   ($3x^2 + 4x$, $5x + 1$)

6) Polynomials with three terms are called **trinomials.**

   ($8x^3 + 5x^2 + 4x$)

7) The highest degree of a variable in a polynomial is called **the degree of the polynomial.**

   Example: In the polynomial $3x^{10} + 5x^2 + 3$, the degree of the polynomial is 10.

8) A polynomial of degree one is called **linear polynomial.**

Example: $x + 1$, $2x + 3$

9) Polynomials of second degree are called **quadratic polynomials.**

Example: $2x^2 + x + 1$, $x^2 + 3$

10) Polynomials of degree three are called **cubic polynomials.**

Example : $x^3 + 2x^2 - x - 1$

11) The real number a, is a zero of the polynomial p(x) if p(a) = 0.

12) Learned to find the remainder by dividing a polynomial by a polynomial.

For example : Divide $(3x^2 + 18x + 5)$ by $(x + 3)$.

$$
\begin{array}{r|l}
 & 3x + 9 \\
\hline
x + 3 & 3x^2 + 18x + 5 \\
 & \underline{-3x^2 \pm 9x} \\
 & \quad 9x + 5 \\
 & \quad \underline{9x \pm 27} \\
 & \qquad -22
\end{array}
$$

Quotient: $3x + 9$

Remainder : -22

Chapter 3

linear equations in two variables

Dear Students! In previous classes we had studied about variables. You will remember– **a quantity whose value keeps changing is called a variable quantity.** Variable quantities are represented by the English alphabets x, y, z….. etc. You must have learned how to write mathematical statements to find the value of an unknown quantity and use variables in puzzles etc. You will remember that in a linear equation with one variable, if the degree of the variable is 1 then it is called **a linear equation.**

For example: If adding 3 to a number gives ten, then find that unknown number.

Suppose the unknown number is x.

Unknown number + 3 = 10

$$\underset{\substack{\uparrow \\ \text{variable quantity}}}{x} + 3 = 10$$

Hence, $x + 3 = 10$ This is called a mathematical statement or equation in one variable, here the degree of variable x is one, hence it is **a linear equation in one variable.**

Then , $x + 3 = 10$

$x = 10 - 3 = 7$

Hence, 7 is the unknown number which when added to 3 gives 10. Let us look at some more examples of linear equations in one variable.

$x + 2 = 8$, $9 + y = 10$

$z - 1 = 9$, $y - 2 = 6$

Remember :- A linear equation in one variable has a unique (one and only one) solution, it is called the root of the equation. An equation in one variable is generally represented as $ax + b = 0$, where a and b are real numbers and *a* is not zero.

❖ Dear Students! Discuss with your teacher why *a* cannot be zero ?

linear equations in two variables -

Let us consider the following situation –

Rashtriya Adarsh Ved Vidyalaya, Ujjain, in all scored 50 runs in their innings in a cricket match. Express this information (puzzle) in the form of an equation.

In this you can observe that the runs scored by either of the two batsmen is not known which means there are two unknown quantities. we are shubham And the runs (unknown amount) made by Rajat can be expressed as x and y and written in the form of an equation. the In this, if we consider the number of runs scored by Shubham is x and the number of runs scored by Rajat is y , Then

(Number of runs of Shubham) + (Number of runs of Rajat) = 50

$$x + y = 50$$

Which is an required equation.

This is an example of a linear equation in two variables.

$$\underset{\text{variables}}{x} + \underset{\text{variables}}{y} = 50$$

In the above equation the variables are denoted by x and y but you can use other letters.

(a, b, …… z) can also be used.

Ruchi : I have written the above equation like this. A + B = 50

Guru ji – Yes , it is absolutely correct, we can represent variables using any letters.

Such equations in which there are two unknown quantities (variables) and the exponent of each variable is one, it is called **linear equation in two variables.** You may remember -

Shubhm: The exponent of x in x^2 is 2, we will read this as x to the power 2. And in, the exponent of x in x is 1, we read it as x^1

Following are some examples of linear equations in two variables.

1) $3x + 4y = 5$ 2) $3m + 5n = 9$

3) $2p + q = 1$ 4) $A + 3B = 6$

Can you give some more examples? Note that you can write these equations respectively as

$3x + 4y - 5 = 0$, $3m + 5n - 9 = 0$,

$2p + q - 1 = 0$ and $A + 3B - 6 = 0$.

A linear equation in two variables can be represented in general form as $ax + by + c = 0$, where a, b and c are real numbers. Both a and b are not zero ($a \neq 0, b \neq 0$) and the exponent of the variables is 1 which we call as **linear equation of two variables.**

Example : From the following equation find the values of a , b and c.

1) $2x + 3y - 5 = 0$

Solution : Compare the given equation $2x + 3y - 5 = 0$ with the general form of linear equation in two variables $\mathbf{ax + by + c = 0}$. When compared to -

$a = 2, b = 3, c = -5$, which are real numbers.

2) $5x + 3y = 4$

Solution : Given: $5x + 3y = 4$

or $5x + 3y - 4 = 0$

the comparing the given equation $5x + 3y - 4 = 0$ with the general form of linear equation in two variables $\mathbf{ax + by + c = 0}$

$a = 5, b = 3$ and $c = -4$.

3) $\sqrt{3}x - 4y = -5$

Solution: Given: $\sqrt{3}x - 4y = -5$

Or $\sqrt{3}x - 4y + 5 = 0$

the given comparing the equation $\sqrt{3}x - 4y + 5 = 0$ with the general form of linear equation in two variables. $\mathbf{ax + by + c = 0}$

$a = \sqrt{3}$, $b = -4$ and $c = 5$.

Write the following statement as a linear equation in two variables.

Statement: The cost of one book and 10 pencils is Rs 50.

Suppose, the price of a book is x and that of a pencils is y, then according to the statement the equation is ,

(Price of book) + 10 (Price of pencil) = 50

Hence , $x + 10y = 50$

Do and Learn –

By comparing the following equations with the general form of linear equation in two variables, find the values of a, b and c.

$4x + 5y + 16 = 0$

a = ……… b = ………. c = ………

Exercise 3.1

1. Select the correct option from the following multiple-choice questions.

(a) Which of the following is a linear equation in two variables?

(I) $2x + 5 = 0$ (II) $x + y = 1$ (III) $y = 2$ (IV) $2x = 3$

(b) Which of the following is a linear equation in two variables ?

(I) $2x + 5 = 0$ (II) $3x + 2 = 0$ (III) $5 = 2x$ (IV) $2x+y=7$

(c) Which of the following is a linear equation in two variables?

(I) $2x = 3$ (II) $y = 2$ (III) $x = 3y$ (IV) $x = -5$

2. Write a linear equation in two variables to represent the following statements.

A) If two balls and one bat cost Rs 200. write a linear equation in two variables.

B) The difference between the costs of 3 upvastras and 2 shawls is Rs. 200.

3. Rewrite the following linear equations in form of $\mathbf{ax + by + c = 0}$ find the values of a, b and c.

1) $5x + 6y = 18$ 2) $7x + 8y + 9 = 0$

3) $10m + 4n + 6 = 0$ 4) $-7m + 5n + 10 = 0$

5) $4y + 10z - 12 = 0$ 6) $4y + 10z + 12 = 0$

Solution of linear equation:

In the following verse, certain points to be remembered before solving linear equations in two variables are given below.

एकाव्यक्तं शोधयेद् अन्यपक्षाद् रूपाण्यन्यस्य इतरस्माच्च पक्षात् ।

शेषाव्यक्तेन उद्धरेद् रूपशेषं व्यक्तं मानं जायते ऽव्यक्तराशेः।।

अव्यक्तानां द्व्यादिकानामपीह ,यात्तावद् द्वयादिनिघ्नं हृतं वा।

युक्तोनं वा कल्पयेद् आत्मबुद्ध्या मानं क्वापि व्यक्तमेवं विदित्वा।

(बीजगणितम्,एकवर्णसमीकरणम्, पृ. 263)

while solving an equation, keep the following things in mind. There is no effect on a linear equation if-

1. The same number must be added or subtracted to both sides of a linear equation.
2. Both sides of a linear equation is multiplied or divided by the same non-zero number.

Let us learn using example.

Example : Which of the following ordered pairs verifies (satisfies) the equation $x + y = 10$?

(1) (4 , 5) (2) (3 , 7) (3) (1 , 9)

Solution 1): (4 , 5)

equation $x + y = 10$

If $x = 4$ and $y = 5$ then substitute the values in the equation

$4 + 5 \neq 10$

$9 \neq 10$

left side $\neq$ right side

Which is not correct as $9 \neq 10$

Here we can write the values of x and y as ordered pair (4 , 5).

Solution 2): (3, 7)

equation $x + y = 10$

Ordered Pair (3 , 7) substituting $x = 3$ any $y = 7$ in the equation

$3 + 7 = 10$

$10 = 10$

left side = right side

Hence, the ordered pair (3, 7) is the solution of the given equation because we know that the left side and right side of the equation are equal.

Solution 3): (1, 9)

the equation $x + y = 10$

In the ordered pair (1 , 9) when the value of x is 1 and the value of y is 9, then substituting the values in the equation.

$1 + 9 = 10$

$10 = 10$

Hence, the ordered pair (1 , 9) is the solution of the given equation $x + y = 10$.

you observe that the equation x + y = 10 is not verified (satisfied) by the ordered pair (4, 5) hence the ordered pair (4, 5) is not a solution.

In contrast , the ordered pairs (3 , 7) and (1 , 9) verify (satisfy) the equation (left side = right side). Hence (3,7) and (1 , 9) are the solutions of the equation. You observe that in this (3 , 7) and (1 , 9) are solutions. Therefore, a linear equation in two variables can have different solutions. This means that **a linear equation in two variables has infinitely many solutions.**

The following question is found in Indian Mathematics on the linear equation in two variables in Algebra, written by Bhaskaracharya:

एको ब्रवीति मम देहि शतं धनेन त्वत्तो भवामि हि सखे द्विगुणस्ततोऽन्य:।

ब्रूते दशार्पयसि चेन्मम षड्गुणोऽहं त्वत्तस्तयोर्वद धने मम किं प्रमाणे।।

(बीजगणित, अनेकवर्णसमीकरण पृ.272)

Two friends are discussing their financial situation. My money will double if you give me Rs 100, said the first friend, then the second friend said, friend! If you give me Rs 10, my money will become six times your money; then calculate how much money they will both have.

Solution: - Suppose the money with the first friend is x and the money with the second friend is y , then according to the question,

$x + 100 = 2(y - 100)$ Equation (1)

$y + 10 = 6(x - 10)$ Equation (2)

Then from equation (1)- $x + 100 = 2(y - 100)$

$x + 100 = 2y - 200$

$x + 100 = 2y - 200$

$x = 2y - 200 - 100$

$x = 2y - 300$ ....Equation (3)

From equation 2- $y + 10 = 6(x - 10)$

$y + 10 = 6x - 60$

$x = \frac{y+70}{6}$ .....Equation (4)

From equation (3) and (4) -

$(2y - 300) = \frac{y+70}{6}$

$6\,(2y - 300) = y + 70$

$12y - 1800 = y + 70$

$12y - y = 70 + 1800$

$11y = 1870$

$y = \frac{1870}{11} = 170$

y= 170

y Substituting the value of in equation (3)-

$x = (2y - 300)$

$x = 2 \times 170 - 300$

$x = 340 - 300$

$x = 40$

That means the first friend has Rs. x That means Rs 40. And the second friend has Rs. y That means Rs 170. There was money.

Exercise 3.2

1. Select the correct option from the following multiple choice questions.

(a) When $x=2$, $y=1$ then the value of k in the equation $2x + 3y = k$ will be-

(I) $k = 5$ (II) $k = 6$ (III) $k = 7$ (IV) $k = 8$

(b) Which of the following is the solution to the equation $2x + y = 7$?

(I) $x = 2, y = 3$ (II) $x = 3, y = 2$ (III) $x = 1, y = 2$ (IV) $x = 2, y = 1$

(c) Which of the following is the solution of the equation $x + 4y = 17$?

(I) $x = 1, y = 4$ (II) $x = 4, y = 1$ (III) $x = 2, y = 1$ (IV) $x = 4, y = 2$

(d) The solution of $x + y = 4$ is -

(I) $x = 1, y = 3$ (II) $x = 0, y = 0$ (III) $x = 4, y = 1$ (IV) $x = 1, y = 4$

(e) The solution of $x + 2y = 6$ is -

(I) (2,2) (II) (0,2) (III) (2,0) (IV) (-2, -2)

(f) Which of the following is not a solution of the equation $y = x + 2$?

(I) (0,2) (II) (1,3) (III) (- 2,0) (IV) (2, 3)

(g) Which of the following is a solution of the equation $3 = 2x + y$?

(I) (0,3) (II) (3,0) (III) (- 1, 4) (IV) (- 2, 7)

(h) Which of the following is a solution of the equation $y = 3x$?

(I) (1, 3) (II) (3,1) (III) (2,5) (IV) (6, 2)

(i) Which of the following is a solution of the equation $y = 2x+1$?

(I) (1,3) (II) (2,4) (III) (3,5) (IV) (4, 6)

2. Which of the following options is correct and why?

$$m + n = 20$$

which of the following equations are solutions of $m + n = 20$.

(1) (10, 5)   (2) (10, 10)   (3) (3, 15)

(4) (2, 12)   (5) (15, 5)   (6) (20, 0)

3. Find the value of K if $x = 30$ or $y = 70$ is a solution of the equation $x + y = K$.

4. Find the value of K if $m = 15$ and $n = 15$ is a solution of the equation $m + n = K$.

5. Find the value of K if $x = 2$, $y = 1$ is the solution of the equation $2x + 3y = K$.

We learned -

1) An equation in which there are two unknown quantities (variables) and the exponent of the unknown quantities is 1, they are called **linear equations having** one or two variables.

- The general form of a linear equation in two variables is $ax + by + c = 0$ where a, b and c are real numbers a and b are not zero ($a \neq 0, b \neq 0$)
- A linear equation in two variables has many solutions in infinite form.
- Learned to write mathematical statements in two variables as linear equations.

Chapter 4

Vedic Mathematics

Utility of Vedic Mathematics

Calculations become short and simple when Vedic mathematical formulas are used. Calculation takes less time as well. Vedic Mathematics helps students develop their mental and logical abilities. Students' interest in mathematics grows as the possibility of error in calculations decreases. Even complex calculations can be solved verbally with a little practice in Vedic mathematics. The simple formulas of vedik mathematics assist students in increasing their self-confidence. As a result, the learners' intelligence and aptitude have greatly improved.

List of sutras and sub-sutras of Vedic mathematics

Formula:

1) Akadhiken Purvene: By one more than the former
2) Nikhilam Navatashcharam Dashat: From all nine to the last ten
3) Urdhvatiryagbhayam: horizontal and cross multiply
4) Paravarty yojayet: Use of opposites
5) Shoonyomyan samyamyasamuchchaye: zero if the set is qual.
6) Anurupya Shunyamanyat: If one is in proportion, the other will be zero.

7) Sakalaan-vakalaanaabhityaam: By adding and subtracting

8) Purnapurnabhyam: From complete and incomplete

9) Chalankalnabhyam: Simultaneous movement

10) Yavadoonam: As small as possible

11) Vyashti Samashti: The whole is the same and like a whole.

12) Sheshaannomyanaken charamen: Remaining from the last digit

13) Sopantanyodvayamntayogyam: By adding end with its double

14) Ekanakyoonen poorven: one less than the previous

15) Gunit samuchchay: Sum of multiplying numbers of the product

16) Gunak samuchchay: Set of factors

Vedic sub-sutra –

1) By analogy: by ratio

2) Shishyate Shesha Sangya: To find the remainder from the remainder.

3) Aadya Ma Dayanantyamantyen: first is from the first and the last is from the last.

4) Kevalai: Saptakam Gunyat: Multiples of only seven

5) Veshtanam: Ashleshan (name of the specific operation of divisibility test)

6) Yaavdunam Taavdunam: As much as you can reduce , reduce it further.

7) Yaavdunam taavdunim kritya vargan cha yojayet: Whatever is less, reduce it by double the number and use square.

8) Antyayordashke: When the sum of the last digits is ten.

9) Antyorev: Only to the last

10) Set Multiplication: Product of sets

11) Lopa Nasthapanabhyam: By lopana and sthapana

12) Vilokanam: Seeing

13) Multiplied Set: Multiplied Set: The set of products is the product of sets.

14) Dwanda yog: sum of compliments

15) Shuddha: Point

16) Dwajank: Unit digit of the divisor

Meaning and applications of special formulas

1) Ekadhikena Purvena Sutra:

This formula means '**one more than the previous**'. To do Akadhiken of a number, add one to it or mark the sign above it.

As shown below.

Akadhiken of 2 $= \dot{2} = 2 + 1 = 3$

Akadhiken of 3 $= \dot{3} = 3 + 1 = 4$

Akadhiken of 3 in $1\dot{3}4$ is $= 144$

Akadhiken of 2 in 6525 is $= 65\dot{2}5 = 6535$

Application –

1) Operation of Addition:

Method: Write the numbers in the question from top to bottom in a column structure. Begin adding from the top of the units column. If the sum is ten or more, write Akadhikena si on the previous digit. Repeat this process until all of the digits of the numbers have been added. The reminder is written in the answer after adding the last digit of the number.

Example : Add:

37383 + 15228+ 34581+ 23773

Solution:

$$
\begin{array}{r}
3\,7\,3\,8\,3 \\
\dot{1}5\,\dot{2}\,\dot{2}\,8 \\
3\,\dot{4}\,5\,8\,1 \\
+\ \dot{2}3\,\dot{7}7\,3 \\
\hline
\boxed{1\,1\,0\,9\,6\,5}
\end{array}
$$

Hint –

1) In the first column, 3 + 8 = 11, hence put Ekadhikena sign on the digit 2 before 8.

2) Units digit of 11, 1 + 1 = 2

Example : Add: 72.86, Rupees 32.25 and Rupees 38.82

Solution: Rupees Paisa

$$
\begin{array}{rcl}
7\,2 & . & 8\,6 \\
3\,\dot{2} & . & \dot{2}\,5 \\
+\ \dot{3}\,8 & . & 8\,2 \\
\hline
1\,4\,3 & . & 9\,3
\end{array}
$$

Hint –

In this also add the respective columns

Operation of Subtraction:

In Vedic mathematics, among the four to five methods of operation of subtraction, the best and simplest method is based on (Ekadhiken Sutra + Paramamitra).

Parammitra Number –

Two numbers whose sum is 10 are called **Parammitra Number** of each other.

Such as : Parammitra number of 7 = 3

Parammitra number of 6 = 4

Paramamitra number of $\dot{2}$ (= 3) = 7

Method- When the lower digit (subtrahand) cannot be subtracted from the upper digit (minuend), the lower digit's parammitra digit is added to the upper digit, and the sum is written below (in the answer), and the previous digit of the lower number is marked with the Ekadhikena sign. We get the answer by repeating the steps.

Example: Subtract the following:

52124 – 27271

Solution : 5 2 1 2 4

- $\dot{2}$ 7 $\dot{2}$ 7 1

$\boxed{2\ 4\ 8\ 5\ 3}$

Hint-

1) 4 – 1 = 3 Write in the answer space below.

2) 7 does not decrease in 2, hence the best digit of 7 is 3. Add 2 to 2 and write the sum 5 below the answer.At place as well as at number 2 before 7 action of subtracting multiple signs in a similar manner Complete it.

Example : subtract:

Solution:

	H	M	S
	3 4	3 2	1 5
-	$\dot{1}$5	$\dot{2}$ $\dot{4}$	3 2
	1 9	0 7	4 3

Hint –

1)The units of measurement (time) in each column are different.

2) There will be two digits in both the minutes and seconds column.

3) Base in the units column of both = 10

4) Base in both the tens column = 6

5) Base of hour column = 10

6) The basis for calculating Parammitra number in the minutes and seconds column will be 6 the base of the reminder is 10.

Operation of Multiplication:

In Vedic mathematics, there are different formula based methods for different situations of multiplication. Though the method based on Sutra Ekadhikena Purvena is effective, some special multiplication methods are very simple and interesting.

the Atyayordashkepi formula, multiplication is done using Ekadhikena.

- This formula has limited use in multiplication, it works only where the sum of the unit digits of the product and multiplier is 10 and the remaining digits are the same, the result is obtained in two parts.
- **Right Side –** Multiply the ones digits and write the product.
- **Left side** - Write the product of (remaining digits of tens or ones) ×(remaining digits of tens or ones + 1).

Example , Multiply 24 by 26.

Solution :

$$\begin{array}{r} 2\,4 \\ \times \quad 2\,6 \\ \hline \boxed{6\,2\,4} \end{array}$$

Hint -

1) Here the sum of the unit digits is 4 + 6 = 10.

2) The tens digits of the multiplier and multiplicand are the same.

Write 4 × 6 = 24 on the right side.

3) 2 × (Ekadhiken of 2)

Example: Multiply 83 by 87.

Solution :

$$\begin{array}{r} 8\,3 \\ \times\ 8\,7 \\ \hline \boxed{7\,2\,2\,1} \end{array}$$

Hint –

1) Here the sum of the unit digits (extremes) 3 + 7 = 10.

2) Remaining tens digit is 8 (same)

Hence, right side = 3 × 7 = 21

3) Left side = 8 × (Ekadhikena of 8)

= 8 × 9 **= 72**

Example : Multiply. (Using Sutra Akiken)

Solution: $3\frac{5}{6} \times 3\frac{5}{6}$

$= 3 \times 4 \ / \ \frac{5}{6} + \frac{5}{6}$

$= 12 \ /\frac{10}{6}$

$= 12 \ \frac{10}{6}$

Hint -

1) sum of fractions

5/6 + 5/6 = (5+5)/6 = 10/6

2) Remaining next digit (Nikhilam digit) = 3

= 3 × (Ekadhikena of 3)

= 3 × 4 = 12

Using this formula, questions on multiplication can be done orally.

Do and Learn – Multiply (Using Ekadhikena Purvena Sutra)

1) 3 4	2) 8 2	3) 9 3
× 3 6	× 8 8	× 9 7
$\boxed{}$	$\boxed{}$	$\boxed{}$

Ekkunnen Purvena Sutra- The **s**utra has two words namely, 'Ekanunen' And 'Purveen'. The meaning of the sutra is: If there is ekkunnen sign (.)the digit with a dot sign at its bottom**.** Subtract 1 from its previous digit. This dot is called Ekanyun mark.

Example: Eknunena of 7 = $\underset{\cdot}{7}$ = 6

Eknunena of 15 = $1\underset{\cdot}{5}$ = 14

Application –

Subtraction problems can be simplified by this method. (Ekanunen Purvena formula + Paramamitra Ank)

Method –

If the separator does not decrease from the separable, add the parametr of the separator to the separable and write the sum at the bottom along with a dot below the previous digit of the separator, this point is called the equinox, the frequency of this operation will determine the end result (answer).

Example : Subtract the following (from Eknunen Purvena formula)

560 – 375

Solution:

	5	6	0	→ separable
–	3	7	5	→ separator
	1	8	5	→ remainder

Hint –

1) based on the Ekadhiken purvena formula

2) The difference is the position of the Ekadhiken sign (at the top) of the digit is placed at the Bottom (Ekanunen sign) of the previous digit.

Do and Learn –

subtraction.(Using Ekanunen Purvena Sutra)

1) 5 8 0
– 3 9 4

2) 7 4 8
– 3 7 9

Operation of Multiplication –

Using Ekanunen Purvena Sutra

This amazing method of multiplication is used, in the multiplication of two numbers, when each digit of 9.

Method : There are two sides to the product.

the left side = multiplier – 1

right side = multiplier– the left side

hence,

multiplicand × multiplier = Multiplicand – 1 / Multiplier –Left Side

This formula works in three situations.

1) **First situations:**

(No. of digits in Multiplier = No. of digits inMultiplicand)

See the following example

Example : Multiply - 8× 9

Solution: the left side = 8 – 1 = 7

right side = 9 – 7 = 2

Hence 8 ×9 = 8 – 1 / 9 – 7

= 7 / 2

= 72

Example : Multiply 345 ×999

Solution : 345 ×999

= 345 – 1 / 999 – 344

= 344 / 655

= 344655

2) **Second situation :**

(No.of digits in Multiplier > No. of digits inMultiplicand)

Example: Multiply - 34 ×999

Solution: 34 ×999

= 034 – 1 / 999 – 033

= 33 / 966

= 33966

Example : Multiply - 254 ×99999

Solution : 254 ×99999

= 254 – 1 / 99999 – 00253

= 253 / 99746

= 25399746

Remember: –

1) Write the given numbers by writing as many zeros as required before the multiplier or multiplicand such that the number of digits on both are same, out of which all the digits of one number is 9.
2) The sum of the remaining digits on the left side and the right side respectively is 9 which means-

   left side First digit + right side digit = 9

3) Third Situation:

(No.of digits in Multiplier < No. of digits in Multiplicand)

Example : Multiply- 53 × 9

Solution: 53 × 9

= (53 – 1) / 9 – 52

= 529/ – 52

$= \quad 477$

Example: Multiply - 312×99

Solution: 312×99

$= \quad 312 - 1 \ / \ 99 - 311$

$= \quad 31199 \ / \ -311$

$= \quad 30888$

Other Method :

If the number of 9's in the multiplier is less than the number of digits in the multiplicand, form a new number by writing at the end of the multiplicand as many zeros as the number of 9's in the multiplier and subtract the multiplicand.

Example : Multiply - 312×99

Solution:

$$\begin{array}{r} 31200 \\ -312 \\ \hline \boxed{= 30888} \end{array}$$

Do and Learn –

Find the products of the following. (using Ekanunen Purvena Sutra)

1) 32×999 2) 3112×99 3) 121×999

4) 2×999 5) 452×99 6) 951×999

Exercise 4.1

1. Fill in the following blanks.

   (a) Meaning of Ekadhikena Sutra is…………………….

   (b) Meaning of Ekanunen Purvena Sutra is…………………….

   (c) Paramamitra digit of number 8 is…………..

   (d) 456×999 = ...

   (e) 456×99999 = ...

   (f) 4568×99 = ...

2. Add using Ekadhikena sutra purvena.

   1) $\begin{array}{r} 98765 \\ 34549 \\ +\ 12757 \\ \hline \boxed{} \end{array}$ 2) $\begin{array}{r} 38219 \\ 21989 \\ +\ 13456 \\ \hline \boxed{} \end{array}$ 3) $\begin{array}{r} 456 \\ 397 \\ +\ 138 \\ \hline \boxed{} \end{array}$

3. Subtract (difference) using the Vedic method.

   1) $\begin{array}{r} 982 \\ -\ 137 \\ \hline \boxed{} \end{array}$ 2) $\begin{array}{r} 747 \\ -\ 388 \\ \hline \boxed{} \end{array}$ 3) $\begin{array}{r} 4037 \\ -\ 2158 \\ \hline \boxed{} \end{array}$

4. Multiply using the formula (Atyodarshkeshpi + Ekadhikena)

   1) $\begin{array}{r} 97 \\ \times 93 \\ \hline \boxed{} \end{array}$ 2) $\begin{array}{r} 58 \\ \times 52 \\ \hline \boxed{} \end{array}$ 3) $\begin{array}{r} 77 \\ \times 73 \\ \hline \boxed{} \end{array}$

5. Multiply: (using eknunena method)

1) 52×99 2) 173×999

3) 47×999 4) 134×9999

5) 72×9 6) 123×99

6. Multiply by Vedic method. (Using ekadhikena purvena Sutra)

1) $2\frac{5}{7} \times 2\frac{3}{7}$ 2) $5\frac{11}{3} \times 5\frac{2}{3}$ 3) $5\frac{5}{3} \times 5\frac{5}{3}$ 4) $6\frac{11}{7} \times 6\frac{2}{7}$

Remembering point :

- **Vinakulam (negative) number -**

Use of Vinakulam is the gift of Vedic mathematics. Calculations become short and simple and sometimes even oral, by Vinakulam, method. In this method, the number greater than 5 (6 , 7, 8, 9) are converted into smaller digits (0 , 1, 2, 3, 4, 5). Which makes calculations easier.

As , $\overline{1}\,\overline{2}, \overline{5}, \overline{7}$small line above the digits is theVinakulam (−) sign.

- **Beejank (numeric) sum) :**

Sum of all the digits of a number is called its beejank. Always the beejank of a number is a single digit number. If the sum of the digit of a number is more than 10, the digits are added to obtain a single digit number, which is the beejank of the given number. **Example:**

(The beejank of 75 is 7 +5= 12, but there are two digits, hence 1 + 2 =3)

While finding the beejank, ' 9 ' is considered equivalent to 0 why ,

Example : The beejank of 531 will be - 5+3 **+** 1 = 9.

beejank of 172654 will be 1 + 7 + 2 + 6 + 5 + 4 = 25 from 25, 2 + 5 = 7

- **Base: -**

In Vedic mathematics, 10,100, 1000 or powers of 10 are considered as the basis to make calculations easy

Remember:–

The number of zeros is in the product is equal to the number of zeros in the base. the product is written as non-zero digits followed by zeros.

For example: In base number 10, 1 digit is written on the right side , in base number 100, 2 digits are written on the right side, in base number 1000, 3 digits are written on the right side.

- **Sub – base: –**

The sub - **base** is a multiple of the base number. mostly it ends with zero. If the base number is a multiple of 10 (20, 30, 40, 50, 60, 70 …………) or the base number is a multiple of 100 (200, 300, 400 …………..) etc. If the base is = 10, then the sub-base is = 10 a, where an is a whole number.

Example:

Base = 20 = 10 × 2

Then , base = 10 and sub – base = 2.

- **Deviation –**

The difference between the number and the base is called its deviation

deviation = number - base

Example:

When the base is 10.	When the base is 100.
Deviation of 18 = 10 + 8 = +8	Deviation of 102 = 102 – 100 = +2
Deviation of 8 = 8 – 10 = – 2	Deviation of 93 = 93 – 100 = – 3
Deviation of 12 = ________	Deviation of 92 = ________

If the number is grater than the base number, **the deviation** is positive. If the number is less than the base number, **the deviation** is negative.

Nikhilam Navatah Charam Dashatah Sutra -

Meaning – This sutra means '**all nine and the last ten.** In ancient Indian mathematics, 9 is called Brahma number and 10 is called full number. This vinakulam sutra is used in many situations related to subtraction, multiplication and division.

Application –

1) Converting general numbers to Vinkulam numbers.

(Ekadhikena Purvena Sutra + Nikhilam Sutra)

When the given number is greater than 5 or 5 It can be converted into Vinkulam number using Nikhilam formula.

Method -

1) Subtract the units digit of the number from 10.
2) Subtract the remaining digit of the number from 9.
3) Draw Vinkulam sign on each digit of the remainder.
4) draw a ekadhiken sign on the first digit of the remainder, 0 or if the digit less than 5.

Let us clarify with examples.

1) Converting general numbers to Vinkulam numbers.

Example : Convert the general number 1 2 7 to Vinkulam number.

Solution: On converting the numbers 1 2 7 into Vinkulam numbers –

$$1\ 2\ 7$$
$$= 1\ \dot{2}\ \bar{3}$$
$$= 1\ 3\ \bar{3}$$

Example : 1 6 8 Convert general number to Vinkulam number.

Solution: On converting the numbers 1 6 8 into Vinkulam numbers –

$$1\ 6\ 8$$
$$= 1\ \dot{6}\ \bar{2}$$
$$= 1\ 7\ \bar{2}$$
$$= \dot{1}\ \bar{3}\ \bar{2}$$
$$= 2\ \bar{3}\ \bar{2}$$

2) Converting Vinkulam number to general number

(Ekanyunen Purvena Sutra + Nikhilam Sutra)

Method -

1) Subtract the positive value of the units digit by 10.
2) Subtract the positive values of the remaining Nikhilam digits (except the ones digits) from 9.
3) Repeat the above procedure when required.

Let us clarify with examples.

Convert the Vinkulam number into the general number

Example : - $1\bar{3}$ Convert Vinkulam number to general number.

Solution: Number $1\bar{3}$ Converting to Vinkulam number

$= 1\,\bar{3}$

$= \underset{\cdot}{1}\,7$

$= 0\,7$

Example: Convert the Vinkulam number $2\,\bar{7}$ Into general number.

Solution: On converting the number $2\,\bar{7}$ into general number

$2\,\bar{7}$

$= \underset{\cdot}{2}\,3$

$= 1\,3$

Example : Convert Vinkulam number $6\bar{2}\,\bar{4}$ into general number.

Solution: Number $6\bar{2}\,\bar{4}$Converting to general number

$= \; 6\,\bar{2}\,\bar{4}$

$= \; \underset{\cdot}{6}\,8\,\bar{4}$

$= \; 5\,8\,\bar{4}$

$= \; 5\,\underset{\cdot}{8}\,6$

$= \; 5\,7\,6$

3) Multiplication of two numbers (Nikilam Navatah Charam Dashatah Sutra)

When two numbers are close to base 10 or 100 or power of 10, then their product can be done very easily on the basis of Nikhilam formula.

Method -

1) Choose the nearest base 10 or 100 according to the numbers.
2) Write the deviations respect to the base in against of the numbers.
3) Divide the product space into two parts with a diagonal line.
4) Write the product of deviations on the right side.
5) one number + another number on the left side.
6) Write on the right side, as many zeros as there are zeros in the base. If there are less number of digits write 0 on the right to complete if the number is more then add it on the left side.

7) If the product of deviations is negative convert it into positive form by taking one from the left side.

Remember : – the value of the one coming from the left side is equal to the base of the right side.

Let us clarify with examples.

Example : Nikhilam Navatah Charam Dashat: Multiply by (base 10 and 100) method.

- **when base is 10**

1) Multiply: 13×12

$$
\begin{array}{rl}
 & 13 + 3 \\
\times & 12 + 2 \\
\hline
 & 13 + 2 \;/\; 3 \times 2 \\
 & 15/6 \\
= & 156
\end{array}
$$

Hence $13 \times 12 = 156$

Hint –

1) Deviation = +3, +2

2) Take 13 + 2 or 12 + 3 on the left side.

3) Multiplication of deviations on the right side

= 6 (one digit)

- **when base is 100**

2) Multiply: 92×93

$$
\begin{array}{rl}
 & 92 \quad -08 \\
\times & 93 - 07 \\
\hline
 & 92 - 7 / (-8) \times (-7) \\
 & 85 \;/\; 56 \\
= & 8556
\end{array}
$$

Hint –

1) Deviation = – 08, – 07

2) Write two digits on the right side, hence 56.

3) On the left side we take 92 – 7 or 93 – 8.

Hence 92 ×93 = 8556

3) multiply : 93 ×102

$$\begin{array}{rr} 93 & -7 \\ \times\ 102 & +2 \\ \hline \end{array}$$

$93 + 2 \ / \ (-7) \times 2$

$\underbrace{95 \ / \ -14}_{+1}$

$94 \ / \ 100 - 14$

$94 \ / \ 86$

$= \quad 9486$

Hence 93 ×102 = 9486

Hint:–

1) product 95 /- 14

2) Bring 1 from the left side to the right side.

3) Place value of 1 on the right side = 100

4) Multiplication of two numbers (Sub-base formula – Nikhilam Navatah Charam Dashatah)

If the deviations of the numbers in a question are large, it becomes difficult to multiply them. In such a situation the concept of sub-base is used. In this the base digit is multiplied on the left side and the right side remains the same as before.

Let us clarify with examples.

Example Multiply by: Nikhilam Navatah Charam Dashatah (sub – base method).

1) Multiply: 32 ×34

$$\begin{array}{rl} & 3\,2 + 2 \\ \times & 3\,4 + 4 \\ \hline \end{array}$$

32 + 4 / 2 ×4

36 ×3 / 8

108 / 8

= 1088

Hence 32 ×34 = 1088

Hint –

1) Base = 10

Base = 3 × 10 = 30

base digit = 3

2) Base deviation = + 2 and + 4

3) Multiplication of base digit 3 on the left side = 36 × 3 = 1088

2) Multiply- 64 ×67

$$\begin{array}{rl} & 6\,4 + 4 \\ \times & 6\,7 + 7 \\ \hline \end{array}$$

(64 + 7) ×6 / 4 ×7

71 ×6 / 28

426+2 / 8

= 4288

Hence 64 ×67 = 4288

Hint –

1) Sub - base = 10 × 6

base digit = 6

2) Multiplication of base digit 6

= 71 × 6 = 426

3) After that the right side should be adjusted.

3) Multiply- 306×312

$$\begin{array}{rl} & 306 \;\; +6 \\ \times & 312 + 12 \\ \hline & 318 \times 3 \,/\, 72 \\ & 954 \,/\, 72 \\ & = 95472 \end{array}$$

Hint –

1) Base = 100
2) sub -base = 100×3

   Sub - Base number = 3
3) Deviation = +6 and +12

Hence $306 \times 312 = 95472$

DIVISION METHODE (Formula Nikhilam Navatah Charam Dashatah) –

Method of writing questions –

Make three sections of the space allotted by drawing two vertical lines, write the divisor in the first section on the left and the complementary number below it. Write as many digits of the dividend as there are zeros in the base, from the unit digit, in the third section. Write the remaining digits of the dividend in the middle section.

Procedure or method –

1) Write the first digit of the dividend from the left in place of the sum.
2) Multiply this digit by its complementary number and write the product below the second digit of the middle section.
3) If there are two digits in the complementary number, then write the product below the third digit also.

4) Find the sum of the digits in the second place and write the result below it.

5) The digits in the third position are not to be added.

6) Then multiply the second digit written in the sum by its complementary number and write the product below the third digit of the dividend and add to it.

Keep repeating this process (repeat), until the digits of the product are written below the units digit of the third section. Add it again at the end. Sum written in middle section = quotient

and the sum written in the third section = remainder.

If the remainder obtained is greater than the divisor, then subtract the divisor from it to get the modified quotient and remainder. Let us clarify with an example.

Example : Find the quotient - 3 7 8 ÷ 8

Solution:

8	1 1	1
2	2	6
	1 3	7
	Quotient	remainder

Hint –

1) Quotient = 13

And remainder = 7

Example : Find the quotient - 1 1 8 9 ÷ 8 8

Solution:

88	1 1	8 9
12	1	2
		2 4
	1 2	13 3
	+1	– 88
	13	45
	Quotient	remainder

Hint –

1) Complement number = 100 – 88 = 12

2) Write down 1 of the middle section 1× The digits of 12 = 12 are written below the next digits in the middle section.

3) Write down 1 + 1 = 2 in the middle section.

4) 2× 12 = 24 Write as shown in middle and third section.

5) Do the sum – Quotient = 12 , Remainder = 133

6) Remainder > divisor

Therefore change is necessary. Then ,

Modified quotient = 13 , remainder = 45

Exercise 4.2

1. Fill in the following blanks.

(a) The general number 46 is written as………… in Vinkulam number.

(b) The general number 28 is written as………… in Vinkulam number.

(c) Vinkulam number $3\bar{3}$is written as ……… in general number.

(d) Vinkulam number $4\bar{4}$is written as ……… in general number.

(e) Deviation of general number 17 =...............

(f) Deviation of number 89 =...............

2. Convert the following general number to Vinkulam number.

A) 89 B) 187 C) 253

3. Convert the following Vinkulam number to general number.

a) $3\bar{2}1$ b) $4\bar{3}2$ c) $5\bar{3}\bar{4}$

4. Multiply by the formula Nikhilam Navatah Charam Dashatah.

a) 103×107	b) 94×95	c) 108×105	d) 96×93
$\boxed{}$	$\boxed{}$	$\boxed{}$	$\boxed{}$
e) 73×74	f) 203×204	g) 506×504	h) 810×804
$\boxed{}$	$\boxed{}$	$\boxed{}$	$\boxed{}$

i) $\begin{array}{r} 302 \\ \times\ 312 \\ \hline \boxed{} \end{array}$	j) $\begin{array}{r} 88 \\ \times\ 83 \\ \hline \boxed{} \end{array}$	k) $\begin{array}{r} 716 \\ \times\ 711 \\ \hline \boxed{} \end{array}$	l) $\begin{array}{r} 407 \\ \times\ 412 \\ \hline \boxed{} \end{array}$

5. Divide by the method of Nikhilam Navatah Charam Dashat :.

a) 1245 ÷ 97 b) 311 ÷ 8 c) 1013 ÷ 88 d) 1113 ÷ 888

We learned

1) Discussed about **16 sutras and 16 sub-sutras** of Vedic mathematics.
2) Solved addition, subtraction, multiplication and multiplication of fractions through formula ekadhikena Purvena.
3) Practiced subtraction and multiplication operations (when two numbers and each digit of the number is 9) through the formula Ekanunen Purvena.
4) Vinakulam: conversion of number to general number and general number to Vinakulam were discussed.
5) Formula Nikhilam using base, sub-base and deviation number learned to multiply by Navatah Charam Dashatah.
6) SutraNikhilam learned to divide by Navatah Charam Dashatah.

Methods of checking answers

Method of checking the answer obtained from any operation in mathematics -

Beejank method:-

To find the beejank of any number, all the digits of that number are added until a single digit is obtained.

Like : **a)** The beejank of 134 is 1+3+4 = 8.

b) The beejank of 78 is 7+ 8 = 15. in this, 15 is obtained which is not the beejank. Hence Adding its digits again 1+5 = 6

- **Checking the result of problem on addition**

  The answer will be correct if the beejank of the result is the sum of beejanks of the numbers added.

Example:

	4 8 1 5	9
	2 4 8 7	3
+	1 9 0 4	5
	9 2 0 6	8

Check: beejank of sum of beejanks of the numbers

$9 + 3 + 5 \longrightarrow 17 \longrightarrow 1 + 7 \longrightarrow 8$

Beejank of the answer

$9 + 2 + 0 + 6 \longrightarrow 17 \longrightarrow 1 + 7 \longrightarrow 8$ both beejanks are equal.

hence, the Answer is correct

Checking the result of the problems on subtraction :

(In this beejank of the number subtracted (second number) + beejank of answer = Beejank of the number from which the second number is subtracted (first number)

Example:

	7 8 1	7
−	3 2 5	1
	4 5 6	6

Check: a) Beejank of second number + Beejank of answer =

$1 + 6 \longrightarrow 7 \longrightarrow$

b) Beejank of first number = 7 $\longrightarrow$

Both beejank are equal, hence Answer is Correct.

- **Checking the result of the problem on multiplication**

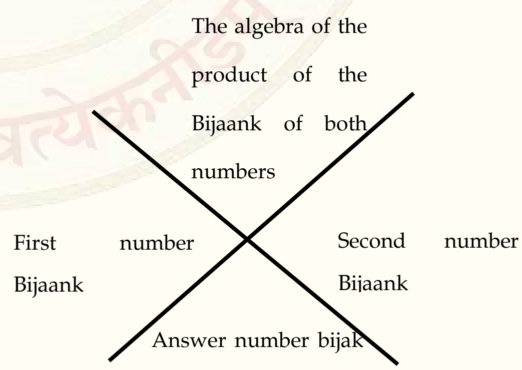
(Beejank of first Number × Beejank of Second Number)

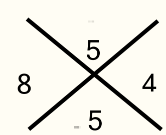
= Beejank of the Product

Example : 4 1 3 ×5 1 7

Solution : 4 1 3 × 51 7

2 8 9 1
4 1 3 0
2 0 6 5 0 0
2 1 3 5 2 1





Check : **a)** Beejank of first Number × Beejank of second Number

Beejank of the Product Of cotyledons ⟶ 8 × 4 = 3 2 Beejank

⟶ 5

b) Beejank of the answer ⟶ 5

Both beejanks are equal is , hence, answer is Correct

- **Checking the result of the problems on division**

Example : 4 8 5 7 ÷1 4

Solution :

Divisor 14) 4 8 5 7 (3 4 6 quotient

$$\begin{array}{r} \underline{-\,4\,2} \\ 0\,6\,5 \\ \underline{-\;\;5\,6} \\ 0\,9\,7 \\ \underline{-\;\;8\,4} \\ 1\;\;3 \end{array}$$ remainder

Check : Beejank of dividend = Beejank of quotient × beejank of divisior + Beejank of remainder.

⟶ 6 (4 × 5) + 4

⟶ 2 0 + 4

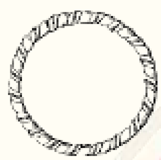
⟶ 2 4

⟶ 6 Which means The Answer is Correct.

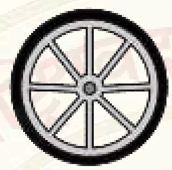
Chapter 5

Circle

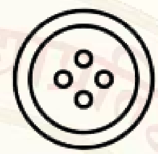
Dear Students! You must have seen or come across with many objects in your daily life, whose shape is in circular form. Such as– Bangles, wheel, buttons of kurta, plate etc.



bangle



wheel



kurta button

Plate

If you turn on a fan in your room, its blades will start rotating rapidly in a circle. In such a situation, instead of seeing the blades separately, you will see a new shape.

You all must have played the game of string and stone? When a string tied to a stone one end the other end is held in the hand and rotated. On increasing the speed of totation, observing the path traced by the ston, you will observe a circular ring form.

A mantra related to calculation of time is given in the following Vedic mantras.

चतुर्भिः साकं नवतिं च नामभिश्चक्रं न वृत्तं व्यतीरवीविपत् ।

बृहच्छरीरो विमिमान ऋक्वभिर्युवाकुमारः प्रत्येत्याहवम् ॥

(ऋग्वेद 1/155/6)

There is reference about circle in the above mentioned text of Rigveda. In which there is an indication of dividing the circle into four parts of ninety degrees each.

In this chapter we will study about terms related to circle.

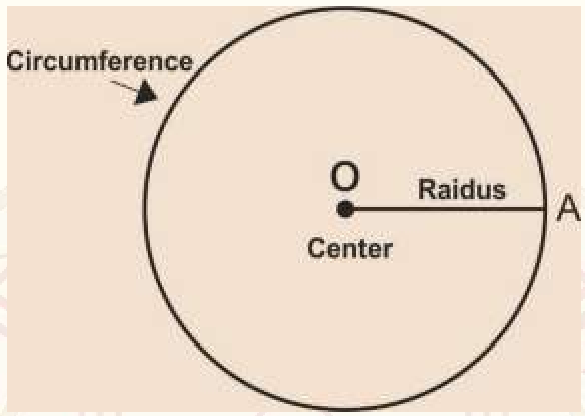
Circle -

Take a compass, and insert a pencil in the pencil holder and keep the pointed end of the compass at a point on the paper (middle of page). Extend the other arm of the compass to some distance, and rotate the other arm once keeping the pointed end fixed at a point. What is the figure made on paper by the pencil ? As you know it is a **circle.** How did you get the circle ?

In reality, a circle is a group of infinite points marked continuously with the tip of a pencil when the compass is rotated, that is, the group of all those points on **a plane which are located at a constant distance from a fixed point on the plane is called a circle.**

Center and radius of the circle –

The fixed point is called the center of the circle and the fixed distance is called **the radius of the circle.** In the figure, O is the center of the circle and OA is the radius of the circle. The length of the entire circle is called **the circumference of the circle.**

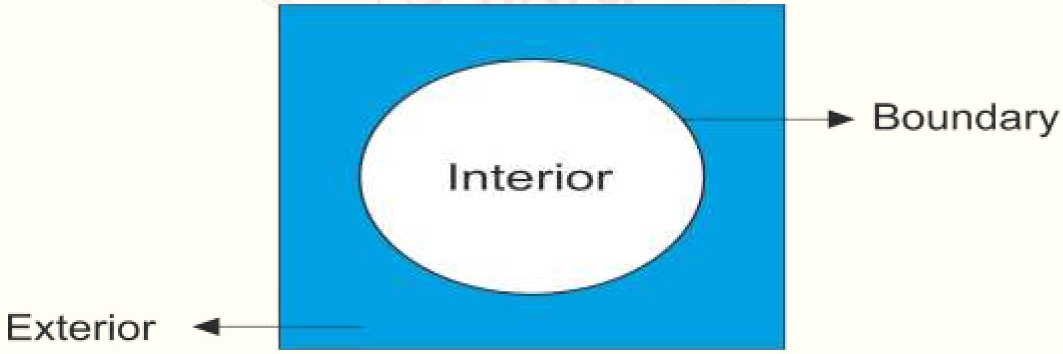


Remember:-

The line segment joining the center of the circle and any point on the circumference is called **the radius of the circle.**

The plane of a circle on which it lies is divided into three parts.

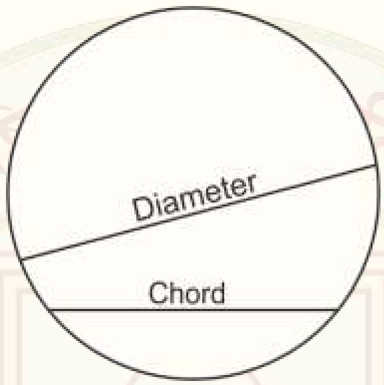
1) **Interior** – The region inside the circle is called interior.
2) **Boundary** – circle (the shape itself)
3) **Exterior** – The region outside the circle is called the exterior.



The circle and its interior together form **the circular region.**

Chord and Diameter –

The line segment obtained by joining any two points P and Q on a circle with the help of a scale $\overline{PQ}$ is called **the chord** of the circle. If a chord passes through the center of a circle, then that chord is called **the diameter of that circle.** As shown : AB



The largest chord of a circle is its diameter. **The diameter of a circle** is twice the measurement of its radius. That means

Diameter = 2× Radius and **Radius =** $\frac{\textbf{Diameter}}{\textbf{2}}$

Example : If the radius of the circle is 10 cm. If so, how long the diameter will be?

Solution : Given that radius of circle = 10 cm.

We know – **Diameter = 2 ×Radius**

= 2× 10 = 20 cm

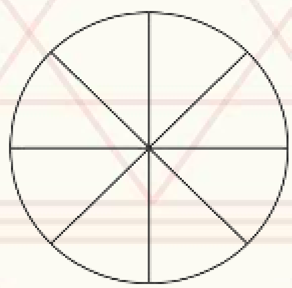
Hence, The diameter of the circle of radius 10 cm will be 20 cm.

Do and Learn –

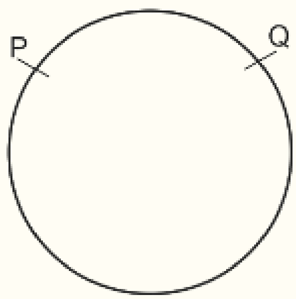
1) In your exercise book, draw circles with different radii and measure the diameter and draw more than two chords in each circle. Measure all the lengths of all the chords ?
2) If the diameter of the circle is 40 cm. What will be the radius of the circle?

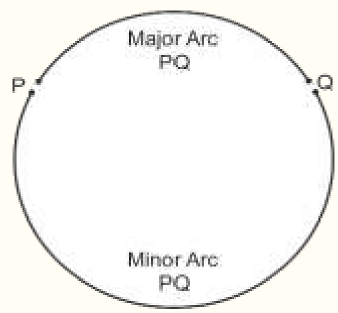
Activity -

Draw a circle of any radius and find the numbr of diameters that can be drawn in it. Will there be more than one diameter ? Yes, infinite number of diameters can be drawn in a circule. See the following figure.



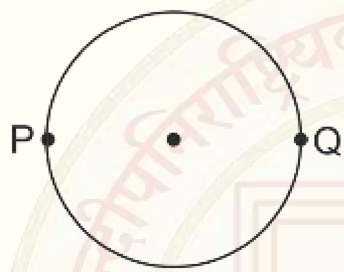
Arc – The circular part between any two points on a circule is called an arc. In the following figure two points P and Q are shown on a circle. Which divides the circle into two parts.

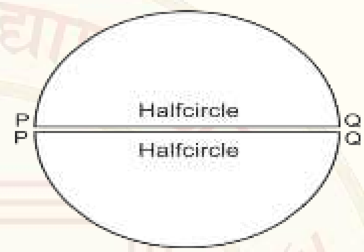




If both the arcs are seen separately in the above figure, then they $(\widehat{PQ})$ are represented by the smaller arc. But the long arc PQ is expressed by taking a point R in the arc $(\widehat{PRQ})$.

If P and Q are located on the diameter and both the arcs are equal then each arc is called **a semicircle.** See the following figure –



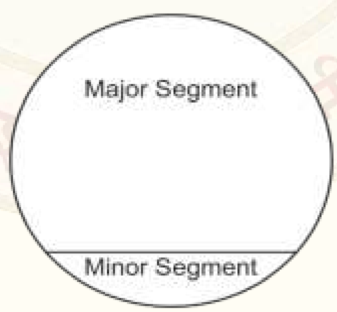


Segment –

A region without a center bounded by a chord and an arc of a circle is called segment. A chord divides the circle into two segments.

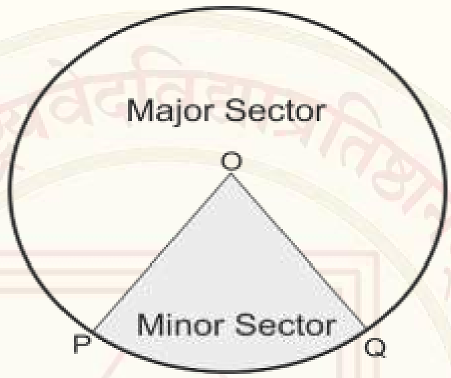
1) major segment

2) minor segment



Sector –

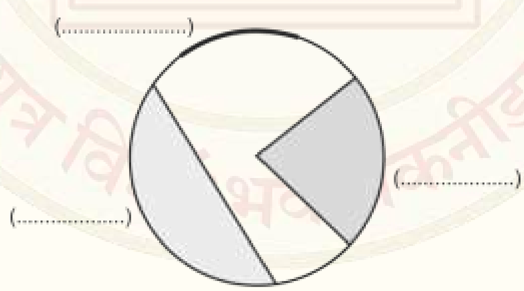
The region enclosed by an arc between any two radii is called sector. In the following figure, OPQ is the minor sector and the remaining is major sectors.



Do and Learn.

Label the following in the figure.

(Arc , Segment, Sector)



Excrcise 5.1

1. Fill in the blanks.

    1) The center of the circle is located in the………… of the circle. (outside/ inside)

    2) A point , whose distance from the center of the circle is more than the radius, is located in the………… of the circle. (outside/inside)

    3) The largest chord of a circle is………… of the circle. (diameter/radius)

    4) An arc is………….. When its ends are the ends of one diameter. (semicircle/circle)

    5) The area between the arc and the chord of a circle is………….. (segment/ sector)

    6) The area enclosed by two radii and an arc is …………………….. (segment / sector)

    7) The distance between the center and circumference of the circle is called…………………….. (chord / radius)

    8) The diameter is the …………… chord of the circle. (smallest / largest)

2. Write true/false.

    1) Circle is a plane shape.

2) The area between the chord and the corresponding arc is the radius.

3) The line segment joining the center to any point on the circle is the radius of the circle.

4) If a circle is divided into three equal arcs, then each arc is a major arc.

5) The largest chord of the circle is its diameter.

6) The chord of the circle on which the center is located is called radius.

7) Infinite number of diameters can be drawn inside the circle.

3. Select the correct option from the following multiple choice questions.

(a) The angle of a semicircle is-

(I) Acute angle (II) Right angle

(III) Obtuse angle (IV) Straight angle

(b) The angles of the same sector are-

(I) Unequal (II) Similar

(III) Both (1) and (2) (IV) None of these

Tangent lines on the circle -

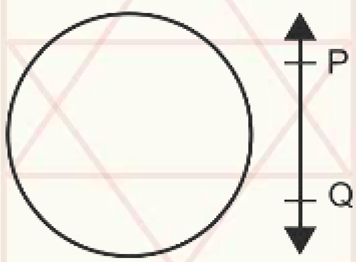
Tangent line-

A coplanar straight line that touches the circle at one point. The circle's tangent line is referred to as the tangent line.

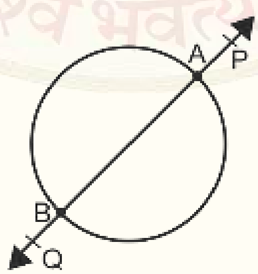
Different positions of a circle and a line situated in a plane:

When a circle and a straight line such as PQ lie in a plane, the following possibilities can occur.

A) Line PQ and circle have no common point. In this case PQ is called **a non-intersecting line** with respect to the circle.

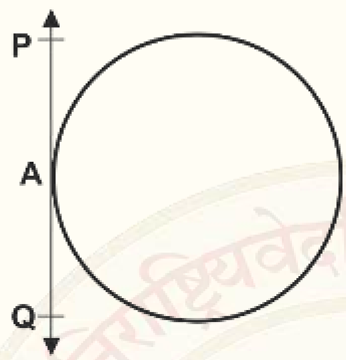


b) Lines P and Q have two common points A and B in the circle. In this case the line is called **secant line of the circle.**



That is, an extended chord which intersects the circle at two points is called **a secant line.**

c) Line PQ has only one common point A in the circle. In this case, line **PQ** is called tangent to the circle.



Tangent line of circle –

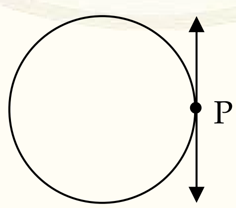
A tangent to a circle is a line that meet the circle at only one point. Tangent to a circle is a special condition of the secant line.

Properties of tangent line –

A) There is one and only one tangent at a given point on the circle.

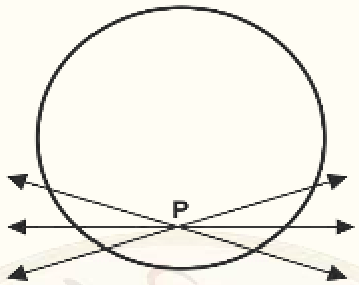
b) The tangent line of a circle is a special condition of the secant line when both the ends of the corresponding chord coincide.

c) The common point of the tangent line and the circle is called point of contact.

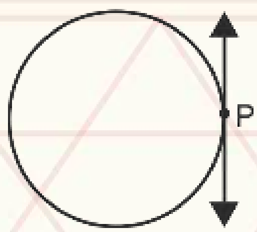


In this, the point P is called the point of contact.

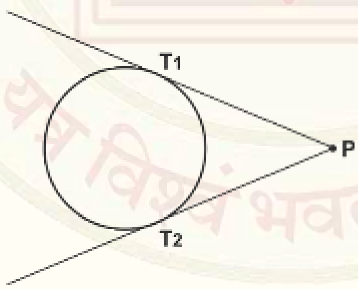
d) There is no tangent to the circle passing through any point lying inside the circle.



e) There is one and only one tangent that can be drawn to the circle from any given point on the circle.



f) There are two and only two tangents to the circle passing through a point lying outside the circle.



In the given figure, the points of contacts of tangents PT_1 and PT_2 Are T_1 and T_2 respectively.

g) The length of the tangent line from the external point P to the point of contact with the circle is called the length of the tangent, from point P to the circle.

Exercise 5.2

1. Select the correct option from the following multiple choice questions.

(a) If ABC is a triangle then how many circles can be drawn from the three vertices A, B and C.

(I) 1 (II) 2 (III) 3 (IV) infinity

2. What is the difference between a tangent line and a secant line of a circle ?

3. Fill in the blanks.

(tangent , secant , one and only one , two and only two , equal , point of contact)

a) An extended chord , which intersects the circle at two points is called ,……………..

b) There is a tangent to the circle from any point on the circle.

c) There are …….. tangent lines to the circle passing from a point outside the circle.

d) The tangents drawn from an external point to the circule,or ……… .

e) The common point of the tangent line and the circle is called………….. .

f) There is one and only one.................... line at a point on the circle.

We learned -

1) A circle is the set of all those points on a plane which are at the same distance from a fixed point on the plane.

2) Terms related to circle –

Center - the point which is at equal distance from all points on the circle.

Circumference- length of curve around the circle is called the Circumference.

Radius- The distance between the center and any point on the circumference of the circle is called radius.

chord- The line segment joining two points on a circle is called a chord.

Diameter - A chord which passes through the center of a circle is called diameter. The diameter is twice the radius.

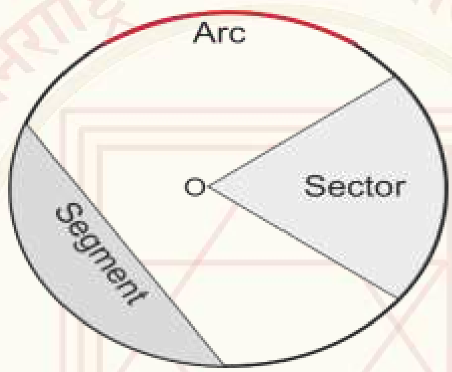
Tangent line- A straight line on a circle which touches the circle at one point is called **the tangent** and the point at which it touches **is called tanget point.**

Secant line – An extended chord that intersects the circle at two points. Is called secant line.

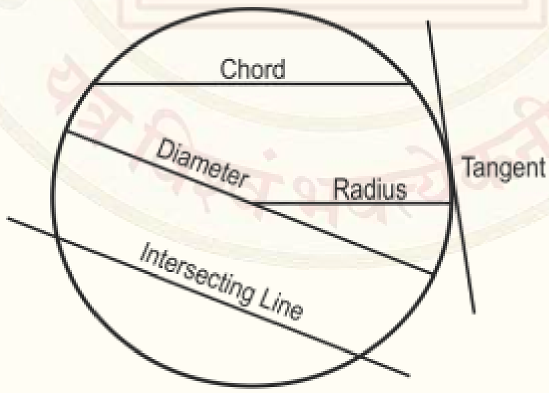
Arc: - Is any part of the circumference of a circle.

Sector: - The region bounded by an arc between any two radii. Is called sector.

Segment :- the region bounded by a chord and an arc is called segment.



3) Studied about the tangent to the circle.



Chapter 6

Congruence and similarity of Triangle

Dear Students, we observe many types of Geometrical shapes in our daily life. Like: Triangle, Quadrilateral, Circle, …... etc. In which many times you observe that the shape and size of any two objects are the same or sometimes the shape is the same but the size is not the same. In this situation will both the figures be similar or identical? In this chapter, we will study in detail the concept of similarity and congruence of triangles.

Revision :

You will remember that a closed shape three sides is called a triangle. the sum of all the three interior angles of a triangle is 180^0. Can you classify the types of triangles (on the basis of sides and angles) ?

Concept of congruence: -

Congruence means- 'Equal in all respects' two figures whose shape and size are identical (same) to each other are called congruent figure, Congruence is represented by the symbol '≅'. You can observe many examples for congruence in your daily life.

Example: 1) Two identica coins of Rs.10

2) Two identical postal stamps.

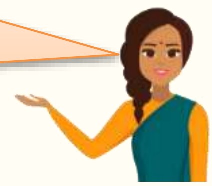
Animesh: If the side of two square shapes is 5 cm, each are congruent to each other?

Congruence of triangles ,

We know that **there are total 6 components of a triangle,namely three sides and three angles.** To check whether two triangles are congruent or not, we need to know about the sides and angles of the triangles. It is necessary that all the six elements of one triangle are equal to all the corresponding six elements of the other triangle, then both the triangles are congruent. In other words we can say two triangles are called congruent triangles, if both triangle cover each other completely on placing one over the other.

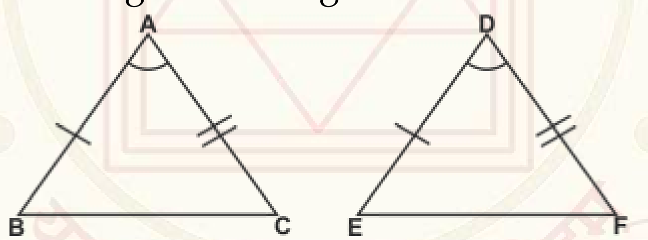
Let us discuss about the rules of congruence of triangle.

Asha: If two triangles have any three elements in common, then both the triangles are congruent ?



Rules of congruence: If the following rules are satisfied by the two triangles, the triangles are congruent to each other.

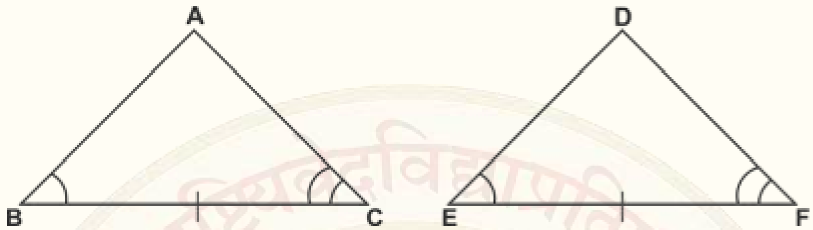
1. **SAS congruence rules (side angle side) :** If any two sides of one triangle and the angle between them are equal to the two corresponding sides of another triangle and the angle between them, the two triangles are congruent.



In the above, triangle ABC and triangle DEF, two pair of sides AB = DE, AC = DF and $\angle A = \angle D$ (angle between the sides) , hence both the triangles are congruent according to the Side – Angle - Side Rule. This congruence can be written as follows .

$$\Delta BAC \cong \Delta EDF$$

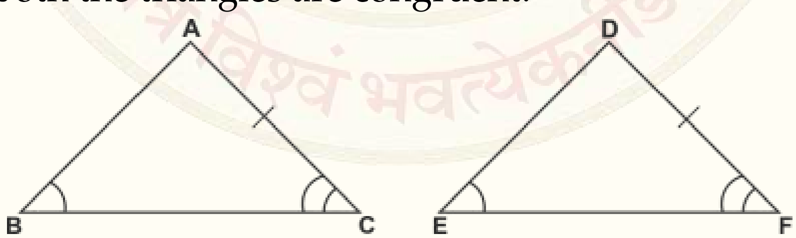
2. **ASA congruence rules (Angle Side Angle) :** If any two angles and the side which includes them of a triangle are equal to the two corresponding angles and one corresponding side of another triangle, the triangles are congruent to each other.



In the above, triangle ABC and triangle DEF, side BC = EF, ∠B = ∠E, and ∠C = ∠F, hence, both the triangles are congruent according to the Angle – side angle rule.

$$\Delta\, ABC \cong \Delta\, DEF$$

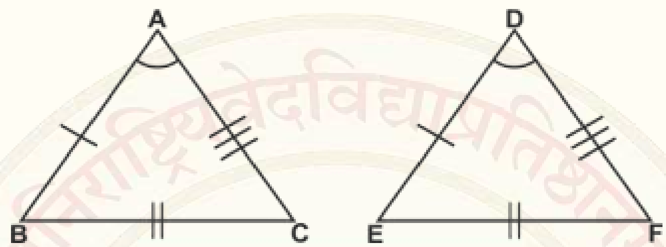
3. **AAS Congruence Laws (Angles angle side) :** Two triangles are congruent if two angles and one side of one triangle are equal to the other. If two corresponding angles and one side are equal, then both the triangles are congruent.



In the above triangle ABC and triangle DEF , side AC = DF, any two pairs of angles are equal ∠B = ∠E and ∠C = ∠F, hence, both the triangles are congruent according to the Angle – Angle - Side rule.

➢ AAS Congruence Rule, is a criterion of ASA rule of congruence.

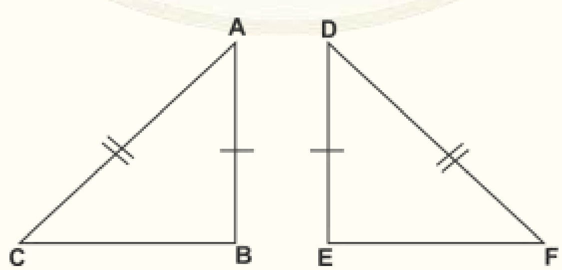
4. **SSS congruence rules (side side side) :** If all three sides of one triangle are equal to the three corresponding sides of another triangle, the triangles are congruent. The pairs of three sides are equal, hence both the triangles will be congruent according to the Side - Side - Side rule.



In the above mentioned triangle ABC and triangle DEF the pair of sides AB = DE, BC = EF and AC = DF Corresponding angle between them $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$ is the same. Hence both the triangles are congruent according to the rule SSS (Side Side Side).

$$\Delta ABC \cong \Delta DEF$$

5. **Rules of RHS Congruence :** If one side and hypotenuse of a right triangle are equal to the corresponding side and hypotenuse of another right triangle, the triangles are congruent.

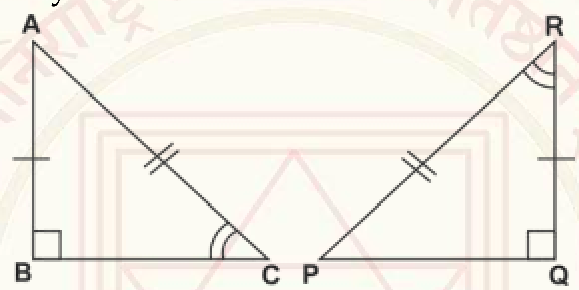


In RHS rule, R stands for right angle, H for hypotenuse and S for side.

In triangle ABC and triangle DEF ∠B = ∠E = right angle (90^0), hypotenuse Side AC = DF and side BC = EF. Hence by RHS rule the triangles are congruent.

$$\Delta ABC \cong \Delta DEF$$

Example : In the following figures, give an additional pair of congruent parts such that ΔABC and ΔPQR are congruent, What congruence rule did you use?



Solution : In this, ΔABC ≅ ΔPQR

∴ ∠B = ∠Q and ∠C = ∠R

∴ Additional pair of congruent parts –

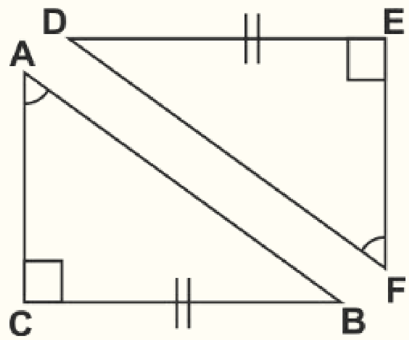
BC = QR

Hence ASA congruence rule is used.

Do and Learn :

Are the following triangles congruent to each other ? if yes, state the rule of congruence.

Δ ABC ≅ ΔFED.



Exercise 6.1 _

1. Fill in the blanks.

   (a) Two line segments are congruent if …………..

   (b) Out of two congruent angles, the measure of one angle is 80° and the measure of the other angle is …………

   (c) If write an angle $\angle A = \angle B$, therefore meaning is both the angle is ....

2. Give two examples which are congruent in our daily life.

3. If Δ ARC $\cong$ Δ FED in pair of scales ABC $\leftrightarrow$ FED, write all the corresponding congruent parts of the triangles.

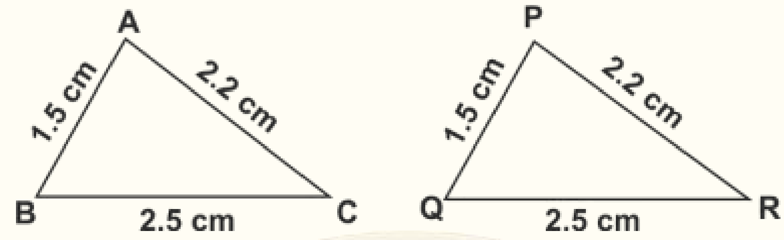
4. If $\Delta ABC \cong \Delta PQR$, write the corresponding parts of ΔPQR of the following :

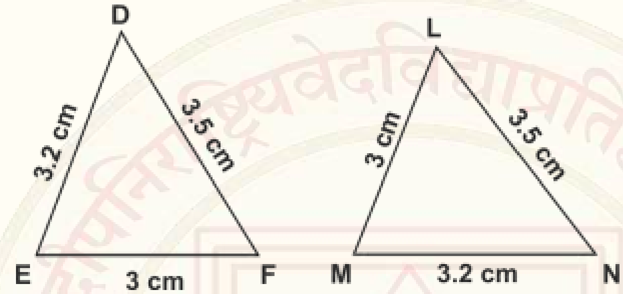
   (i) $\angle B$ (ii) $\overline{BC}$

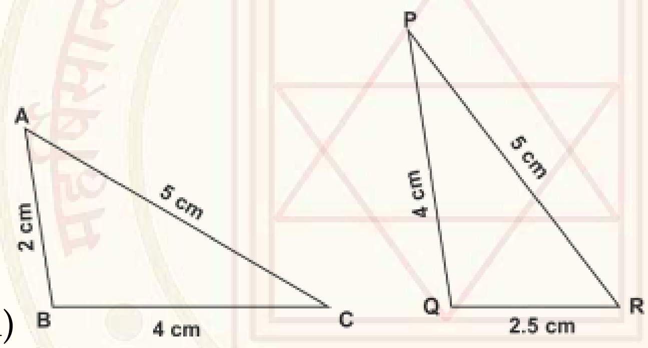
   (iii) $\angle C$ (iv) $\overline{AC}$

5. The lengths of the sides of triangles are shown in the following figure. Using SSS congruence rule find the pair of congruent

triangles. In case of congruence, write the answer in symbolic form.

(i) 

(ii) 

(iii) 

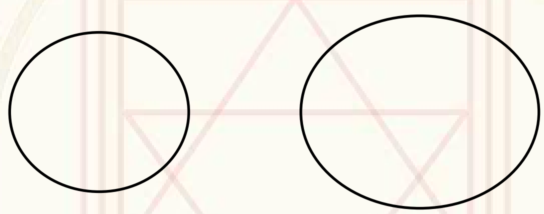
Concept of Similarity :

Students ! Many objects of the same shape but different sizes can be found around us. For example, if you look at the leaves on a tree, you will notice that while their shapes are similar, their sizes are not. Similarly, photographs made from the same negative in different sizes have the same shape; that is, two objects with different sizes but the same shape are called similar figures, and this property is known as similarity.

Let us understand the concept of similarity using the following examples.

1. Two line segments of equal length are congruent and similar but two line segments of different lengths are similar but not congruent.

2. Two circles of the same radius are congruent and similar, but circles with different radii are similar and not congruent.



3. Two equilateral triangles of different sides are similar but not congruent.

It is clear from the above examples that two figures which have the same shape and size are called congruent figures. And two figures which have the same shape but not the same size. Such figure are called similar figure.

Similar Triangles:

You will recall that a triangle is the polygon formed by the least number of sides. We can also write the following condition for the similarity of triangles. That is, two triangles are similar, If -

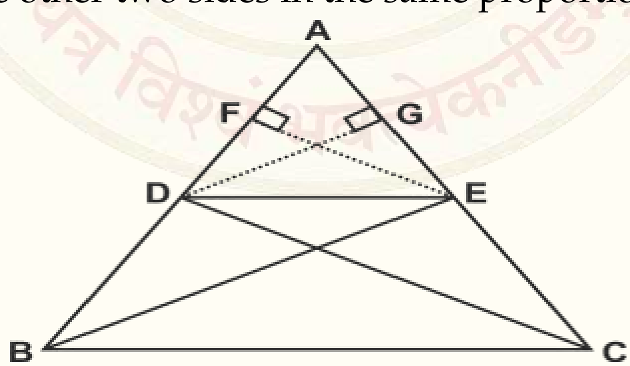
a) Their corresponding angles are equal ,

b) And their corresponding sides are in the same ratio (Which means proportional).

Remember that- if the corresponding angles of two triangles are equal So they are called equiangular triangles. Mathematician Thales has propounded an important fact related to two right angle triangles which is as follows.

Theorem : Write and prove the basic proportionality theorem (Thales theorem)..

statement : In a triangle,the line segment drawn parallel to one of the sides divides the other two sides in the same proportion.



Given : In Δ ABC , DE ∥ BC and DE intersect sides AB and AC at D and E respectively.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE and CD and draw DG ⊥ AC and EF ⊥ AB.

Produce : Δ ADE A and In Δ DBE -

Area of triangle = $\frac{1}{2}$× base × height

Area of triangle ADE = $\frac{1}{2}$× AD × EF …… (i)

Area of triangle DBE = $\frac{1}{2}$ × DB × EF …… (ii)

On dividing equation (1) by equation (2) -

$$\frac{\text{Area of trangle ADE}}{\text{Area of trangle DBE}} = \frac{(\frac{1}{2} \times AD \times EF)}{(\frac{1}{2} \times DB \times EF)}$$

$$\frac{\text{Area of trangle ADE}}{\text{Area of trangle DBE}} = \frac{AD}{DB}\text{……….(iii)}$$

Now in Δ ADE and Δ ECD ,

Area of trangle ADB = $\frac{1}{2}$× AE × DG ………. (iv)

Area of trangle ECD= $\frac{1}{2}$× EC × DG ……… , (v)

On dividing equation (iv) by equation (v) -

$$\frac{\text{Area of trangle ADE}}{\text{Area of trangle ECD}} = \frac{(\frac{1}{2} \times AE \times DG)}{(\frac{1}{2} \times EC \times DG)}$$

$$\frac{\text{Area of trangle ADE}}{\text{Area of trangle ECD}} = \frac{AE}{EC}\text{……….. (vi)}$$

$$\frac{\text{Area of trangle ADE}}{\text{Area of trangle DBE}} = \frac{AE}{EC}\text{……….. (vii)}$$

Note: Area of the triangles drawn between any two given parallel line having the same base are equal.

From equation (iii) and equation (vi) ,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence the basic proportionality theorem $\frac{AD}{DB} = \frac{AE}{EC}$ Proved.

Exercise 6.2

1. Fill in the following blanks.

(a) All circular shapes are ……... (congruent,similar)

(b) All square shapes are…………... (similar, congruent)

(c) All …………. triangles are similar. (isosceles , equilateral)

(d) Two polygons with the same number of sides are similar, if

(i) Their corresponding angles are ………….. and

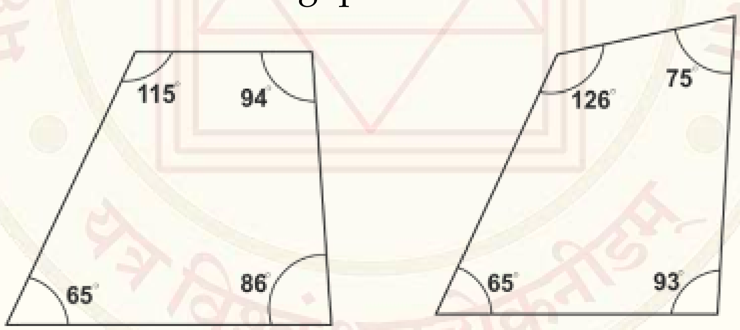
(ii) Their corresponding sides are ………….. (equal, proportional)

2. Give two different examples of the following pairs,

(a) Similar shapes

(b) Figures which are not similar.

3. State whether the following quadrilaterals are similar or not.

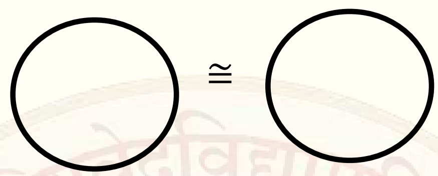


4. Write the statement of basic proportionality theorem.

We learned:

1. Two figures which are same in shape and size are congruent figures. congruent is represented by the ' ≅'.

   Example: Two circle of same radii.



2. Two triangles are congruent to each other, if their three corresponding sides and three corresponding angles are equal.

   Rule of Congruence -

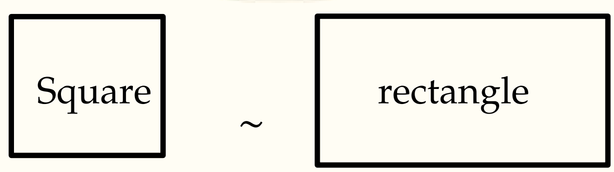
   1. SAS (Side-Angle-Side) Rule 2. SSS (Side - Side - Side) Rule

   3. ASA (Angle-Side-Angle) Rule 4.AAS(Angle-Angle-Side) Rule

   5. RHS (Right Angle-Hypotenuse-side) Rule

3. Two figures which are same and shape but different in size are called similar figures. The symbol of similarity is '~'.

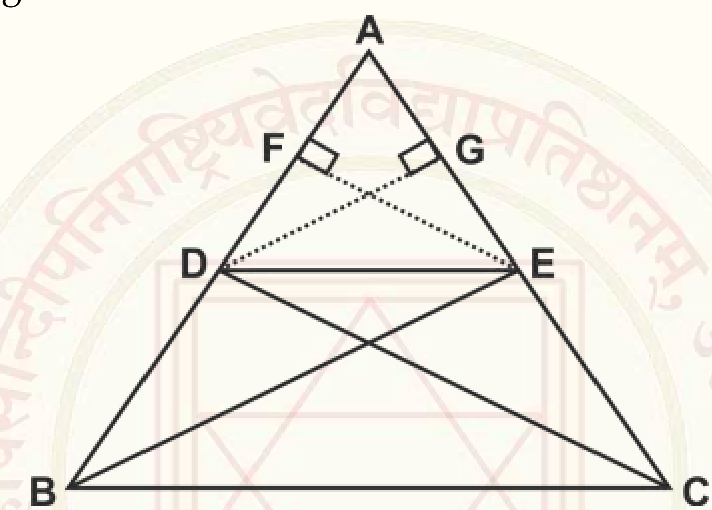
   Example: Two squares of different sizes.



4. **Statement of Thales theorem**,

In a triangle, the line segment drawn parallel to one of its sides divides the other two sides of the triangles in the same proportion.

in triangle ABC-



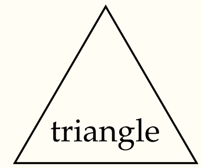
In the above triangle ABC , when side DE is parallel to side BC, then DE intersects sides AB and AC at D and E respectively. Then the following result is obtained. Which is also called basic proportionality theorem.

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Chapter 7

Baudhayana (Pythagoras) theorem

Dear students! We have studied about triangles in previous classes. You will remember that a closed plane figure having three sides is called **a triangle.**



The method of constructing a triangle has been explained in the following mantra of Yajurveda.

तिरश्चीनो विततो रश्मिरेषामधः स्विदासीश्दुपरि स्विदासीश्त् ।

रेतोधाऽआसन्महिमानऽआसन्त्स्वधाऽअवस्तात्प्रयतिः परस्तात् ॥

(यजुर्वेद : 33/74)

The above mantra explains the movment of Sun in all diarection which indicates the formation of a triangle. A triangle is made up of three angles and three sides. Do you know how many types of triangles are there on the basis of angles ?

Triangle Based on angles -

(1) **Acute angled Triangle -** The triangle in which all three interior angles are acute (less than 90^{0}) is called **an acute angled triangle.**

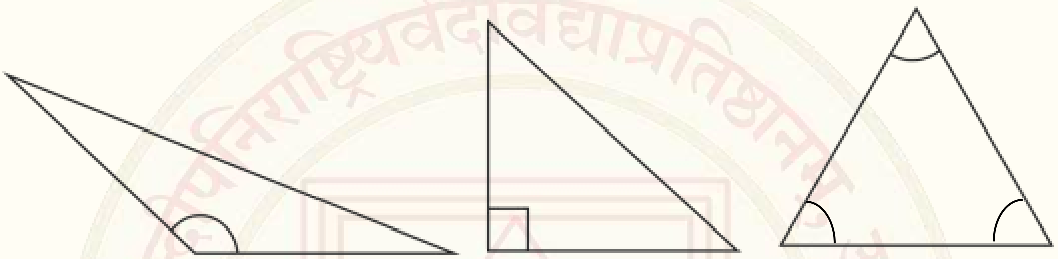
(2) **Right angled Triangle -** The triangle in which one of the angles is a right angle (90^{0}) is called **a right angled triangle.**

(3) Obtuse angled Triangle - A triangle whose one angle is an obtuse angle (more than 90^0) and the remaining two angles are obtuse angles is called **an obtuse triangle.**

Do and Learn

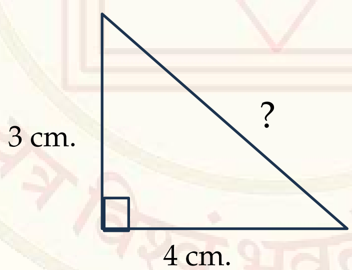
Write the names of the following triangles.

(Right angled triangle , acute angled triangle , obtuse angled triangle)



(……………)　　(……………)　　(……………)

Vishal is curious about right triangles, whose measure of two sides are given and he wants to find the measure of the third side.

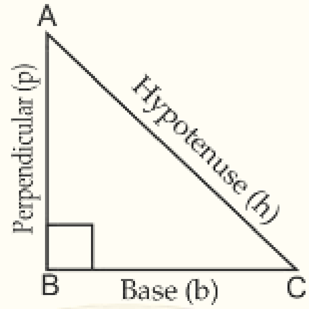


He goes to his teacher and discusses hisn problem.

Guruji - We can find the third side of a right angled triangle when two sides are given, by the theorem given by the famous Indian mathematician Baudhayan ji.

Let us learn, in this chapter we will study in detail the theorem given by **the Indian mathematician Rishi Baudhayan.**

Introduction to right angled triangle –



In the above figure, triangle ABC is a right angled triangle in which the perpendicular and base two sides together form a right angle (90^0). In the triangle ABC angle B is (90^0) (right angle).

Hypotenuse :

- In a right triangle, the side opposite to the right angle is called **the hypotenuse.**
- The hypotenuse is the longest side of a right angled triangle.
- The length of the hypotenuse is less than the sum of the lengths of the remaining two sides, the hypotenuse is often represented by the English letter "h".

Base :

- The lower side other than the hypotenuse which acts as the base is usually called **the base.**
- The side adjacent to an acute angle of a right triangle is called base. (**For example** : The adjacent side of ∠C is the base.)
- Base is often represented by the English letter "b".

Perpendicular - _

- In a right triangle, the side opposite to an acute angle is called **perpendicular.** (**As** , ∠'A 's The adjacent side is perpendicular.)
- Perpendicular (height) is also called perpendicular and is often denoted by the English letter "p".

Note - The side opposite to 90^0 in right angled triangle is called the **hypotenuse.** the other two sides are called base and perpendicular. In a right angled triangle, the angle between the base and perpendicular is (90°) right angle.

Try –

1) In a right angled triangle, the length of the hypotenuse is _____than the sum of the lengths of the other two sides. (less/more)
2) The largest side of a right angled triangle is________ (perpendicular / hypotenuse)

Maharishi Baudhayan –

Baudhayana was an ancient mathematician of India and the author of Shulba sutra and Shrauta sutra. The Pythagorean theorem related to right angled triangle was first given by Maharishi Baudhayana, 460 years before Pythagoras (540 BC), Baudhayana (1000

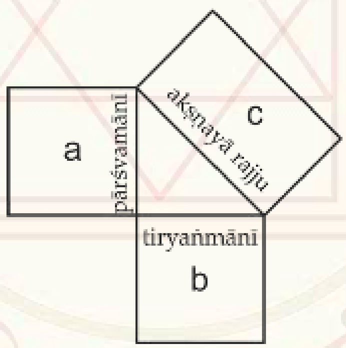
BC) had fully propounded the above mentioned theorem. This is the following sutra of Baudhayana.

सूत्र – **दीर्घचतुरश्रस्याक्ष्णयारज्जुः पार्श्वमानी तिर्यङ्मानी च यत्पृथग्भूते कुरुतस्यदुभयं करोति ।**

(बौधायन शुल्बसूत्र 1.48)

Meaning, if a rope is stretched on the hypotenuse of a right angled triangle, then the area of the square formed is equal to the sum of the areas of the square formed on the vertical and horizontal sides.

In addition to Baudhayana's Shulb sutra, also there is a mention about the theorem in **Manav Shulb sutra 10.10 and Ath Kshetravyavahar 2(A) of Lilavati Mathematics.**



Baudhayan theorem –

The sum of the squares of the sides lengths of the two shorter sides of a right angled triangle is equal to the square of the length of the hypotenuse.

There is a theorem that establishes a relationship between the three sides of a right angled triangle. This theorem is usually expressed as an equation in the following way.

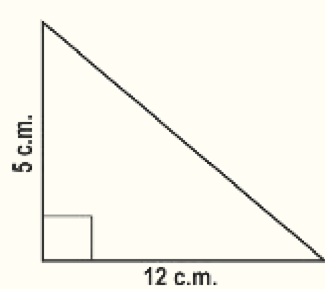
(Hypotenuse) ^{2} **= (Perpendicular)** ^{2} **+ (Base)** ^{2}

$$c^2 = a^2 + b^2$$

or $$c = \sqrt{a^2 + b^2}$$

Where c is the length of the hypotenuse of the right angled triangle, a and b are the lengths of the other two sides. With the help of the relationship between the three sides of this right angled triangle, we can calculate the length of the third side of any right angled triangle, If the length of the remaining two sides are given. Traditionally, the credit for the discovery of this theorem is given to the Greek mathematician Pythagoras, while there is evidence that the theorem was known before that. **This theorem is given in the ancient Indian text Baudhayana's Shulba sutra.** There are other evidences that show even the Babylonian mathematicians know about this thorem. This is also called **Baudhayan (Pythagoras) theorem**. Let us learn, how to solve problems using examples.

Example : In a triangle ABC, angle B is a right angle if the perpendicular is 5 cm. and base 12cm. Find the length of the hypotenuse of the triangle.



Solution: From the figure on the left, since the triangle is a right triangle, hence by Baudhayana (Pythagoras) theorem

$$\textbf{(Hypotenuse)}^2 = \textbf{(Perpendicular)}^2 + \textbf{(Base)}^2$$

$$(AC)^2 = (AB)^2 + (BC)^2$$

or $\quad (AC)^2 = (5)^2 + (12)^2$

or $\quad (AC)^2 = 25 + 144$

or $\quad (AC)^2 = 169$

or $\quad AC = \sqrt{169}$

$$AC = 13 \text{ cm}$$

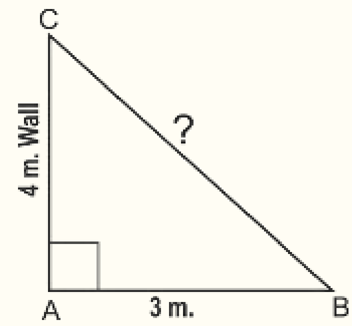
Hence, AC = 13 cm. Thus, the measure of hypotenuse of the given triangle ABC is 13 cm.

Example: A ladder is kept on a wall such that its base is 3m away from the wall and the other and is at 4 m from the floor on a window. Find the length of the ladder.

Solution : Let AC be the ladder and AB be the wall. on which window is at A.

In this base BC = 3 m. And perpendicular AB = 4 m.

Then, from Baudhayana's theorem -



$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

$(AC)^2 = (AB)^2 + (BC)^2$

Or $(AC)^2 = (4)^2 + (3)^2$

or $(AC)^2 = 16 + 9$

or $(AC)^2 = 25$

Or $AC = \sqrt{25} = 5$ m.

Hence the length of the ladder is 5 m.

Example : If the length of the hypotenuse in a right angled triangle is 13 cm. and perpendicular 5 cm. find the length of the base.

Solution : Given: Hypotenuse in a right angled triangle = 13 cm., perpendicular = 5 cm.

Then , from Baudhayana's theorem -

$(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$(13)^2 = (5)^2 + (\text{base})^2$

or $169 = 25 + (\text{base})^2$

or $169 - 25 = (\text{base})^2$

or $(\text{base})^2 = 144$

or base $= \sqrt{144} = 12$

Hence the length of the base is 12 cm.

Example: Solve the following example question based on Baudhayana theorem given in Lilavati Mathematics.

कोटिश्चतुष्टयं यत्र दोस्त्रयं तत्र का श्रुतिः।

कोटिं दोःकर्णतः कोटिश्रुतिभ्यां च भुजं वद।।

In a right angled triangle if we know the base and the perpendicular find the hypotenuse. If perpendicular = 4 and the base = 3 (Any unit of measurement can be used in the above question.)

Solution: Students should solve it themselves.

Do and Learn –

1) Baudhayan's theorem -

(Hypotenuse) ^{2} = (..........) ^{2} + (Base) ^{2}

2) In a right angled triangle, the measurement of the hypotenuse is.....................than the measurement of the perpendicular and base. (less/ more)

Exercise 7.1

1. Select the correct option in the following.

(a) In a right angled triangle, if one side making the right angle is 6 and the hypotenuse is 10, then what will be the length of the other side?

I) 8 II) 10 III) 12 IV) 6

(B) The Pythagorean theorem can be applied only to ________ triangles.

I) Isosceles II) Right angle

III) Isosceles and right angles IV) Equilateral

(c) The largest side of a right angled triangle is…………..

I) Hypotenuse II) Base III) Length IV) none of these

2. Write the statement of Baudhayana theorem.

3. Fill in the following blanks.

(right angle , hypotenuse , $(\text{base})^2$, base , equal , perpendicular)

a) In a right angled triangle, the side opposite to the right angle is called…………. Which is the longest side of a right angled triangle.

b) In a right angled triangle, except the hypotenuse, the other two sides, the perpendicular and the base, together form an angle of _____.

c) Baudhayan theorem –

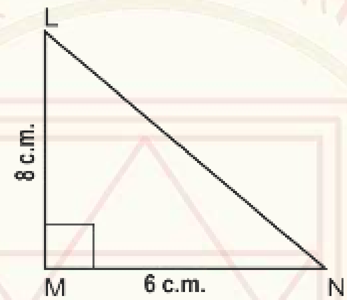
$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + \ldots\ldots\ldots$

d) The horizontal side adjacent to the right angle is called…………..

e) The vertical side adjacent to the right angle is called………….

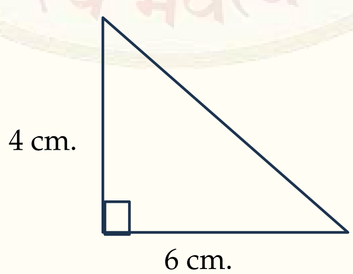
f) In a right angled triangle, the square of the largest side is ……….to the sum of the squares of the remaining two sides.

4. In triangle LMN, angle M is at right angle. If perpendicular (LM) = 8 cm. And base (MN) = 6 cm, find the length of the hypotenuse.



5. In a right angled triangle, the length of the hypotenuse is 25 m. And the length of the perpendicular is 24 m, find the length of the base.

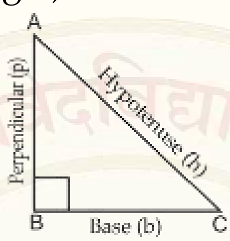
6. In a right angled triangle PQR, right angle is at Q. If PQ = 4 cm and QR = 6 cm Find the value of $(PR)^2$.



7. In a Right angle triangle ABC, side AB = 12 m, BC = 5 meters And angle ABC = 90° Find the measure of side AC.

We Learned -

1) **Right angle triangle** – The triangle in which one of the angles is 90^0 which means (right angle) is called **a right angle triangle.**



2) The side opposite to the right angle is called **the hypotenuse.** This is the longest side of the right angle triangle , it is denoted by "h".

3) The horizontal side adjacent to the acute angle of a right angled triangle is called **base**, it is denoted by "b".

4) The vertical side adjacent to the acute angle of a right angled triangle is called **perpendicular**, it is denoted by " p".

5) A special relationship between the sides of a right angled triangle is known by the name of Baudhayana (Pythagoras) theorem.

6) **Baudhayan's theorem**: The sum of the squares of the lengths of the two sides of a right-angled triangle is equal to the square of the length of the hypotenuse. In other words:

Square of length of hypotenuse = square of length of perpendicular + square of length of base

$$\textbf{(Hypotenuse)}^2 = \textbf{(Perpendicular)}^2 + \textbf{(Base)}^2$$

Chapter 8

Heron's formula

My dear students, We know that in a plane, the shepe bounded by three lines is called **a triangle**, the part enclosed by a closed shape is called its **area** In previous classes, we have learned to find the area of many plane figures (triangle, quadrilateral, circle etc.). In this chapter we will learn to find the area of a triangle using Heron's formula.

Let us repeat the exercise done earlier.

Area of triangle –

In the mantra of Rigveda, (1/105/17) in Vedic literature A triangular shape is mentioned in the form of a word **'Trita'**.The area of a triangle is described in the following mantra found in Atharvaveda in interesting manner.

यो अक्रन्दयत् सलिलं महित्वा योनिं कृत्वा त्रिभुजं शयानः।

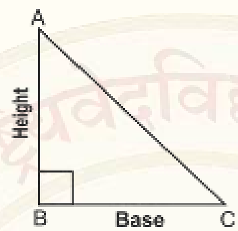
वत्स कामदुघो विराजः स गुहा चक्रे तन्वः पराचैः।

(अथर्ववेद 8/9/2)

Meaning in that area all the characteristics of the area covered under triangular shapes are present. This means that area exists within a triangle and the basis of line mathematics is based on the property of area Which means area. triangular Δ The area enclosed by the shape is called **the area of the triangle.**

Remeber: The units of base and height of a triangle are written as meter (m.) or centimeter (cm) etc. The unit for measuring the area of a plane triangle is taken as square meter (m^2) or square centimeter $(cm)^2$ etc.

Right angle triangle -



$$\textbf{Area of triangle} = \frac{1}{2} \times \textbf{Base} \times \textbf{height}$$

We can find the area of a right angle triangle by the above formula, or this formula is used when the base and height of the triangle are known.

Example : Find the area of the triangle if the height is 4 cm. and base is 5 cm.

Solution: Given: base = 5 cm. , height = 4 cm.

we know,

$$\textbf{Area of triangle} = \frac{1}{2} \times \textbf{Base} \times \textbf{Height}$$

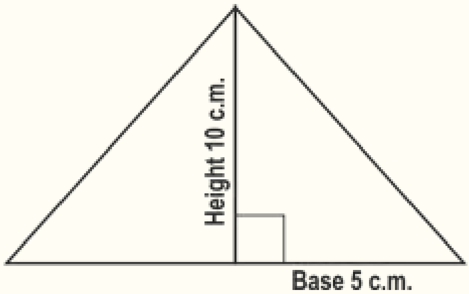
$$= \frac{1}{2} \times 5 \text{ cm} \times 4 \text{ cm.}$$

$$= \frac{20}{2}$$

$$= 10 \text{ cm}^2$$

Therefore, the area of the triangle is 10 cm^2.

Example: Find the area of the following figure.



Solution: The given figure is a triangle.

In this, base is 5 cm. and height is 10 cm.

Area of triangle = $\frac{1}{2}$ **× Base× height**

$= \frac{1}{2} \times 10 \times 5$

$= \frac{50}{2}$

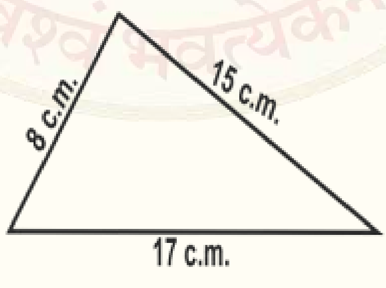
$= 25$ cm.^{2}

Do and Learn –

Find the area of the triangle whose base is 4cm and height is 6cm.

Heron's formula –

Pratyush wants to find the area of a trianglura sheet whose three sides are 8 cm. 15 cm And 17 cm.



How will you find its area ? Of course using the formula - I

$\left(\textbf{Area of triangle } = \frac{1}{2} \times \textbf{ Base} \times \textbf{ height}\right)$If you want to find the area using, you will have to find its height but we do not find its height because it is a **scalene** triangle.

Pratyush and his classmates, this sheres the problem with Guruji.

Guruji – When the length of all three sides of a triangle are known, we find the area of the triangle by Heron's formula. Heron's formula is a special case of Brahmagupta's formula. If the sides of a cyclic quadrilateral are a, b, c, and d the formula to calculate its Area is given by the following verse.

स्थूलफलम् त्रिचतुर्भुज बाहु प्रतिबाहु योग दल घातः।

भुज योग अर्ध चतुष्टय भुज ऊन घातात् पदम् सूक्ष्मम् ॥

(ब्राह्मस्फुटसिद्धान्त,गणिताध्याय, 12.21)

$$\sqrt{\boldsymbol{s}\ (\boldsymbol{s}-\boldsymbol{a})(\boldsymbol{s}-\boldsymbol{b})(\boldsymbol{s}-\boldsymbol{c}))(\boldsymbol{s}-\boldsymbol{d})}$$

Heron's formula is a special case of Brahmagupta's formula when d = 0. Because, if one side of a quadrilateral becomes zero the quadrilateral becomes and every triangle is ' cyclic '. (A circle can be drawn through all three vertices of the triangles.)

सर्वदोर्युतिदलं चतुःस्थितं बाहुभिर्विरहितं च तद्वघात्।

मूलमस्फुटफलं चतुर्भुजे स्पष्टमेवमुदितं त्रिबाहुके।।

(लीलावती गणित क्षेत्रव्यवहारः पृ. 217)

Meaning, by placing the sum of all the sides of a triangle or quadrilateral in three or four places (sides) respectively, subtracting each side from them and taking the root of the product of all the remaining ones, the area of the triangle or quadrilateral is obtained.

If the sides of a triangle are a, b and c respectively then-

Area of triangle = $\sqrt{\boldsymbol{s\ (s-a)(s-b)(s-c)}}$

where s = semiperimeter and a, b and c are the sides of the triangle.

Where **s =** $\frac{a+b+C}{2}$ = $\frac{\textbf{perimeter of the triangle}}{2}$ **= semi-perimeter of the triangle**

Pratyush tries to find the area after Guruji tells him the formula.

Example : The sides of the triangular sheet are 8 cm., 15 cm., 17 cm. find the area of the sheet.

Solution : given:- triangle of sides a **=** 8 cm , b= 15 cm. , c= 17 cm.

semiperimeter of triangle (s) = $\frac{a+b+C}{2}$ = $\frac{8+15+17}{2}$ = $\frac{40}{2}$ = 20 cm.

Now (s – a) = (20 – 8) = 12cm

(s – b) = (20 – 15) = 5cm

(s – c) = (20 – 17) = 3 cm

Area of triangular sheet = $\sqrt{\boldsymbol{s\ (s-a)(s-b)(s-c)}}$

$= \sqrt{20 \times 12 \times 5 \times 3}$

$= \sqrt{3600}$

Or $= \sqrt{36 \times 100}$ Or $\sqrt{36}\ \sqrt{100}$

= 60 square cm.

Example : The sides of a triangular field are 70 meters. 80 m and 90 m. respectively The semi-perimeter of the field is 120 m., then find the area of the triangle.

Solution : Given: semi-perimeter of the triangle S = 120 m.

Let the sides of the triangle be a = 70 m. , b = 80 m. , c = 90m.

Area of triangular field = $\sqrt{\boldsymbol{s}\,(\boldsymbol{s}-\boldsymbol{a})(\boldsymbol{s}-\boldsymbol{b})(\boldsymbol{s}-\boldsymbol{c})}$

$= \sqrt{120(120-70)(120-80)(120-90)}$

$= \sqrt{120 \times 50 \times 40 \times 30}$

$= \sqrt{7200000}$ Sq.m.

$= \sqrt{12 \times 12 \times 5 \times 10000}$ S q.m.

= 1200 $\sqrt{5}$ square meters.

Example : Find the semi-perimeter (s) of the triangle whose sides are 5 cm., 12 cm. and 13 cm.

Solution : Given: Sides of the triangle are a = 5 cm , b = 12 cm. , c = 13 cm.

Semiperimeter of triangle (s) = $\boldsymbol{\frac{a+b+c}{2}}$

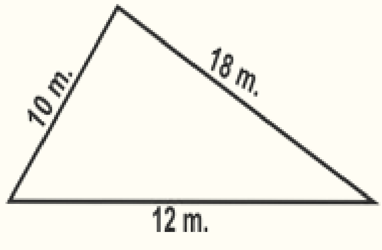
$= \frac{5+12+13}{2}$ cm.

$= \frac{30}{2}$ cm.

= 15 cm

Hence, semi-perimeter of the triangle is 15 cm.

Example – Find the semi-perimeter of the following figure.



Solution : The given figure is a triangle whose sides are 12 m. , 18 m. And 10 m. then we know-

Semiperimeter of triangle (S) = $\frac{a+b+c}{2}$

$= \frac{12+18+10}{2}$meter

$= \frac{40}{2}$ meter

= 20 meters

Exercise 8.1

1. Select the correct option from the following multiple choice questions.

(a) In Heron's formula, area of triangle= $\sqrt{s(s-a)(s-b)(s-c)}$ where a,b And c is-

(I) sides of a triangle (II) Perimeter of triangle

(III) Semi-perimeter of a triangle (IV) Area of a triangle

(b) Heron's formula is –

(I) $\sqrt{(s-a)(s-b)(s-c)}$ (II) $\sqrt{s(s-a)(s-b)(s-c)}$

(III) $\frac{-a+b+c}{2}$ (IV) None of these.

(c) The sides of a triangle are 40 m, 24m and 32m, then the perimeter of this triangle is-

(I) 45 m (II) 96m (III) 24m (IV) 32m

(d) The sides of a triangle are 40 m, 24m and 32m, then the semi-perimeter of this triangle is -

(I) 4 8 m (II) 96m (III) 24m (IV) 32m

(e) The sides of a triangle are 15 m, 11m and 6m then the area of the triangle is-

(I) $10\sqrt{2}\ m^2$ (II) $20\sqrt{2}\ m^2$ (III) $30\sqrt{2}\ m^2$ (IV) $40\sqrt{2}\ m^2$

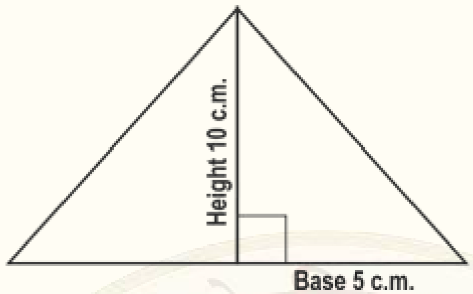
2. Find the area of the following triangles-

$$\left(\textbf{Area of triangle } = \frac{1}{2} \times \textbf{ Base} \times \textbf{ Height}\right)$$

a) base =20 cm. and height 3 cm.

b) base = 4 cm. and height 5 cm.

c) Find the area of the following figure.



3. Find the semi-perimeter of triangles.

$$\left(\text{semi perimeter}(\boldsymbol{s}) = \frac{\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}}{\boldsymbol{2}}\right)$$

4. Find the area of the triangle by Heron's formula.

Heron's formula -

$$\left(\text{area of triangle} = \sqrt{s\,(s-a)(s-b)(s-c)}\right)$$

a) Find the area of the triangle whose semiperimeter is 16 cm. and sides are 8 cm. , 11 cm. And 13 cm.

b) Find the area of the triangle whose sides are 6 cm. , 10 cm. and 14 cm, and the semi-perimeter of the triangle is 15 cm.

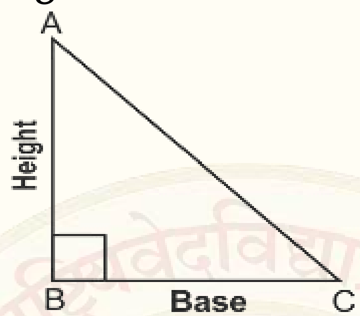
c) Find the area of an equilateral triangle whose three sides are 10 cm each, and semi-perimeter of the triangle is 15

d) Find the area of an isosceles triangle whose two equal sides are 8 cm and the third side is 4 cm.

We learned -

1) If the base and height of a triangle are given then **-**

Right angle triangle -



$$\textbf{Area of triangle} = \frac{1}{2} \times \text{Base} \times \text{height}$$

2) If all three sides of a triangle are given then the area of the triangle is determined by Heron's formula. If the sides of a triangle are a , b and c.

By Heron's formula –

$$\text{Area of triangle} = \sqrt{\boldsymbol{s}\,(\boldsymbol{s}-\boldsymbol{a})(\boldsymbol{s}-\boldsymbol{b})(\boldsymbol{s}-\boldsymbol{c})}$$

where s = semi-perimeter of the triangle.

$$s = \frac{\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{C}}{\mathbf{2}} = \frac{\text{Perimeter of the triangle}}{\mathbf{2}} = \textbf{ the semi-perimeter of the triangle.}$$

Chapter 9

Surface area and volume of cube and cuboid

Dear Vedik Students! we learnt the method of finding area of Triangle, Quadrilateral and Circles in our previous class. In this chapter, we will learn about the method to find surface area and chalkbox, etc.

Brick

Matchbox

Choc Box

Suitcase

These objects are three - dimensional objects, hence, they have length, breadth and height. Thus, they have definite surface are and volume.

Surface area: Surface area of any solid shapes is the sum of the areas of all its surfaces.

Volume: The amount of space occupied by any solid shape is called its **volume.**

Remember: area is measured in square units and volume in cubic units.

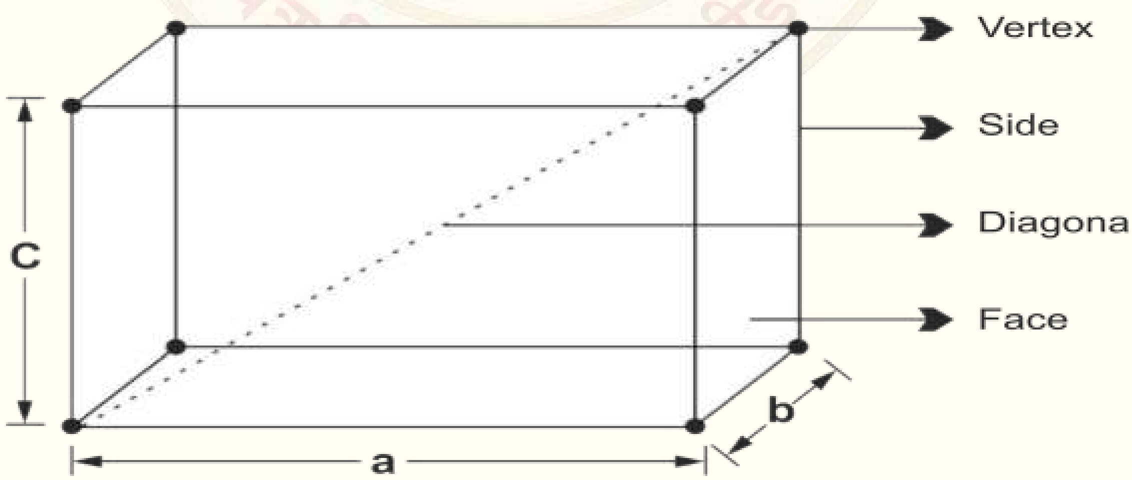
❖ Students! Discuss with your teacher. Why is area measured in square units and volume in cubic units?

Cuboid (Rectangular Solid):

You must have seen rectangular solids like match box, brick, book etc. How many rectangular flat surfaces are there in these objects? In cuboid there are three pairs of identical and parallel surfaces. Parallel surfaces are rectangular and shape. The dimensions of every surfaces are different. A cuboid has length , breadth and height, hence, it is a three - dimensional shape.

Properties of Cuboid:

1) All the surfaces are rectangular.
2) Cuboid has 8 vertices (corners), 12 edges (line segments joining the two adjacent vertices), 6 faces (Which means 3 pairs) and 16 diagonals.
3) Every surface is parallel to the surface opposite it.



In a cuboid, opposite surfaces are identical. To find the surface area of a cuboid, we have to find the area of its six faces and add.

1) **Surface area of cuboid = 2 [ab + bc + ca] square unit**

2) **longest diagonal of the cuboid =** $\sqrt{a^2 + b^2 + c^2}$

Where a = length, b = width , c = height.

Example question –

Example 1 : Length of a room is 3 m, width is 2 m. and height is 4 m find the total surface area of the room.

Solution : Length of the room = 3 m, width = 2 m. , height = 4 m.

Total surface area of the room =

= 2 [length× width + width× hight + hight× length ,

$= 2 [3 \times 2 + 2 \times 4 + 4 \times 3]$

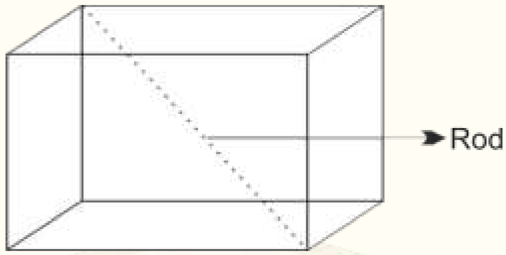
$= 2 [6 + 8 + 12]$

$= 2 [26]$

= 52 square meters

Hence, the total surface area of the room is 52 square meters.

Example 2 : In a box of dimensions 30 cm, 20 cm. and 10 cm, find the length of the longest rode that can be kept in it.



Solution : Length of the box = 30 cm. , width = 20 cm. , height = 10 cm.

The maximum length of the rode that can be kept in it equal to the longest diagonal of the box.

Hence the length of the rode

Diagonal of box (cuboid) = $\sqrt{\boldsymbol{a^2 + b^2 + c^2}}$

$= \sqrt{30^2 + 20^2 + 10^2}$

$= \sqrt{900 + 400 + 100}$

$= \sqrt{1400}$ cm.

$= 10\sqrt{14}\ cm$

Example 3 : If the length of a cuboidal box is 2 m., width 2 m. And height 1 meter. Then find the surface area of the cuboid.

Solution : Length of cuboid = 2 m. , width = 2 m. , height = 1 m.

we know,

surface area of cuboid

= 2 [length× width + width× height + height × length]

= 2 [2 ×2 + 2 ×1 + 1× 2]

= 2 [4 + 2 + 2]

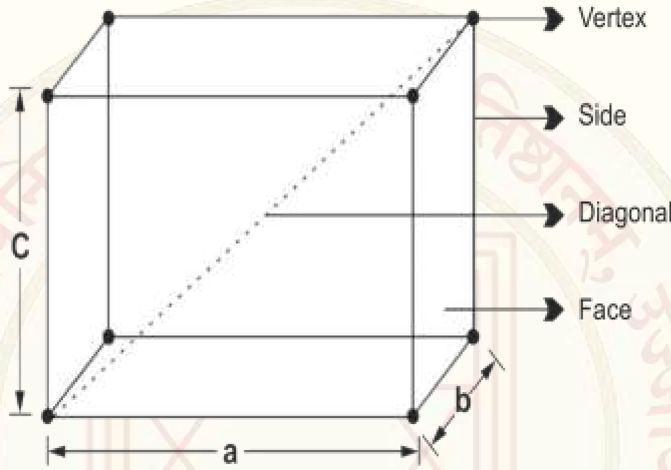
$= 2 [8]$

$= 16$ sq.m.

Therefore, the surface area of the box is 16 square meters.

Cube –

a = length , b = width and c = height.



If the length , breadth and height of a cuboid are equal, it is called a cube. Implies

length (a) = width(b) = height(c) = a (one side)

In this situation, all the surfaces are equal, square in shape, and one side (a) is called a edge of the cube.

length of the longest diagonal of a cube:

$$\text{Longest Diagonal of cube} = \sqrt{a^2 + b^2 + c^2}$$

$$= \sqrt{a^2 + a^2 + a^2}$$

$$= \sqrt{3a^2}$$

$$= a\sqrt{3}$$

Longest Diagonal of cube = $a\sqrt{3}$ Unit

or **= side $\sqrt{3}$ Unit**

Surface Area of Cube:

In a cube, as length, width and height are equal, all the surfaces are square and shape, hence, they have equal area.

thus,

$a = b = c = a$ (one edge of the cube)

Total surface area of the cube = 6× area of a face

$= 6 \times (\text{arm} \times \text{arm})$

$= 6\,(\text{edge})^2$

$= 6\,(a)^2$ square units

Example: An edge of a cube is 2 m. Find the total surface area of the cube.

Solution : Given: One edge of the cube = 2 m.

Therefore,

Total surface area of the cube = 6 (edge)^2

$= 6\,(2)^2$

$= 6\,(4)$

$= 24$ sq.m.

Example : The edge of a cube is 2 cm, find the length of the longest diagonal of the cube.

Solution: Given: edge of the cube a = 2 cm.

Hence, longest **diagonal of cube =** $\mathbf{a\sqrt{3}}$

$$= 2\sqrt{3} \text{ cm}$$

Hence, the length of the longest diagonal of the cube is $2\sqrt{3}$ cm

Example: If Longest Diagonal of a cube is $12\sqrt{3}$ cm. find the edge of the cube.

Solution : Given: Longest Diagonal of cube = $12\sqrt{3}$

Then we know,

Longest Diagonal of cube = $\mathbf{a\sqrt{3}}$

$$12\sqrt{3} = a\sqrt{3}$$

$$a = 12 \text{ cm}$$

Hence, the edge of the cube is 12 cm.

Exercise 9.1

1. Select the correct option for the following multiple-choice questions.

(a) A cube whose edge is a. Total surface area of the cube =......... square unit.

(I) a^2 (II) $4a^2$ (III) $6a^2$ (IV) a^3

(b) If there is a cube whose edge is a , then the longest diagonal of the cube = ...

(I) $a\sqrt{3}$ (II) $a^2\sqrt{3}$ (III) $\frac{\sqrt{3}}{a}$ (IV) $\frac{a}{\sqrt{3}}$

(c) A cuboid whose length , breadth and height are a, b. and c respectively the total surface area of the cuboid = ... square units.

(I) 2 [ab + bc + ca] (II) 2 [ab + bb + ca]

(III) 2 [a + b + c] (IV) $\sqrt{a^2+b^2+c^2}$

(d) A cuboid whose length, Width and height are a, band c respectively.

The length of the longest digonal is =...........................

(I) $\sqrt{a^2+c^2}$ (II) $\sqrt{a^2+b^2}$

(III) $\sqrt{a^2+b^2+c^2}$ (IV) None of these

- **cuboid:**

2. Length of a cuboidal box is 3 m , width is 2 m. and height 3 m. Then find the surface area of the box ?
3. The length, breadth and height of a cuboid are 2 cm. 1 cm. and 4 cm respectively. Find the surface area of the cuboid ?
4. Length , width and height of a room are 5 meters, 4 m. and 3 m respectively. Find the surface area of the room, length of the longest bamboo that can be placed in the room (longest diagonal of the cuboid)?
5. Find the longest diagonal of the cuboid if length, breadth and height are 1 cm. 2 cm. and 3 cm respectively.

- **Cube –**

6. The length of a cubical chalk box is 4 cm. If so, find the total surface area of the chalk box ?
7. If the edge of a cube is 3 m. Then find the surface area of the cube and the longest diagonal of the cube ?
8. The diagonal of a cube is 10 $\sqrt{3}$cm. Find the length of the edge of the cube ?
9. The length of a cubical tank is 2 cm., find the total surface area of the cubical tank ?

Volume of cube and cuboid:

We know that every solid object occupies space, the measure of this occupied space is called the volume of that object. If the object is hollow then it can be filled with fluid or air. The shape of the liquid becomes the shape of the object (vessel), in this situation the amount of liquid filled inside the vessel is called its **capacity.** In Lilavati Mathematics, there is a formula to find the volume of cube and cuboid. As given below.

क्षेत्रफलं वेधगुणं खाते घनहस्तसंख्या स्यात् ।

(लीलावती गणित अथ खातव्यवहार: पृ. 303)

Meaninh , to find the volume of a square shape with any depth Which means cube and cuboid, the volume is obtained by multiplying the area and the (length × breadth) of the shape and its height. In other words, the volume is obtained by multiplying the length, width and height of a cube or cuboid.

The capacity of an object is the volume of the liquid filled inside that object. Its unit is cubic unit. The volume of cube and cuboid can be determined by the following formula.

Volume of cuboid = length × Width ×Height

$= a \times b \times c$

Volume of cuboid = abc cubic unit

Volume of cube:

Breadth and Height of the cube are equal.

Hence, in the volume of the cube a = b = c = a (same edge)

Hence volume of cube = $a \times a \times a$

Volume of cube = a^3 cubic unit

Unit related to volume:

The amount of water that can be store in a water tank shaped cube or cuboid can be found easily.

1 liter = 1000 cubic cm.

1000 liters = 1 cubic meter = 1 kilo liter

1 cubic cm = 1000 cubic milli meters.

1 cubic meter = 1000000 cubic cm.

Example question -

Example: The length, breadth and height of a cuboid are 15 cm × 12 cm × 10 cm respectively Find the volume of the cuboid.

Solution : **Volume of cuboid = length× Width× height**

= 15 ×12 ×10 cubic cm

= 1800 cubic cm. or 1800 $(cm)^3$

Example : A cube has a edge of 5 m, find the volume of the cube.

Solution : Given: One side (edge) = 5 m.

Then **volume of cube =** $\boldsymbol{a^3}$

$= 5^3$ cubic meters.

$= 5 \times 5 \times 5$ cubic meters.

$= 125$ cubic meter

Example : There is a cuboidal water tank whose length is 600 cm.width 500 cm. and depth 200 cm. How many liters of water can it hold?

Solution : Given: Length of cuboidal tank = 600 cm.

Width = 500 cm.

Depth (Height) = 200 cm.

Volume of tank (cuboid) = length× Width ×depth(height)

$= 600 \times 500 \times 200$

$= 6{,}00{,}00{,}000$ cubic cm.

1 liter = 1000 cubic cm

$\frac{1}{1000}$ liter = 1 cubic cm

$= \frac{6{,}00{,}00{,}000}{1{,}000}$ liter

= 60,000 liters

Therefore, 60,000 liters of water can be filled in the tank.

Example : A cubical tank has a length of 2 m. Then how much water should be filled so that it is full in the cubical tank?

Solution : Given: One side (edge) of the cubical tank = 2 m.

Volume of cubical tank = a^3

$= 2^3$

$= 2 \times 2 \times 2$

$= 8$ cubic meters.

We know, 1 cubic meter. = 1000 liters

8 cubic meters = 8×1000 liters

= 8000 liters

Therefore, 8000 liters of water can be filled in a cubical shaped tank.

Example: Volume of a tank is 8000 cubic cm, find the amount of water filled in the tank.

Solution: Given: Volume of tank = 8000 cubic cm.

1000 cubic cm = 1 liter

1 cubic cm = $\frac{1}{1000}$ liter

8000 cubic cm. = $8000 \times \frac{1}{1000}$ liter = 8 liters

Hence, the tank is filled with 8 liters of water.

Do and Learn – Complete the following blanks.

1) 9000 cubic cm. = …………… liter

2) 5000 cubic cm. = …………… liter

3) 3 cubes m. = …………… liter

4) 4 cubic meters. = ……………. liter

Exercise 9.2

1. Find the volume of the cuboid? Whose length, width and height are as follows.

   A) Length = 3 m. , Width = 4 m. , Height = 2 m.

   B) Length = 1 cm. , Width = 5 cm. , Height = 3 cm.

   C) Length = 4 m. , width = 2 m. , Height = 3 m.

   D) Length = 4 cm. , Width = 5 cm. , Height = 6 cm.

2. Find the volume of the cube whose one side or one edge is as follows.

   a) 4 m. b) 5 m. c) 6 cm. d) 10 cm.

3. Length , width and Height of a water tank respectively are 1 meter × 2 m.× 4 m.

   a) How many cubic meters of water can be filled in the tank ?

   b) How many liters of water can be filled in the tank ?

4. A rectangular water tank is 8 m.× 7 m.× 1 m. Is filled up to the brim.

   a) How many cubic meters of water can be filled in the tank ?

   b) How many liters of water can be filled in the tank ?

5. The edge of a cubical tank is 100 cm. then find how many liters of water can be filled in the tank ?

6. If the volume of a cubical tank is 150 cubic meters, how many liters of water will the cubical tank hold ?

7. Measure the length, breadth and hight of the water tank present in you Gurukul and find its volume, hence, find the number of litres of water that it can hold.

We Learned:

1) If the length of the cuboid is a , width is b and height is c ,

Total surface area of cuboid = 2 [ab + bc + ca]

Volume of cuboid = a x b x c

Longest Diagonal of cuboid = $\sqrt{\boldsymbol{a^2+b^2+c^2}}$

2) If one side of the cube is a

Total surface area of cube = $6a^2$

Volume of cube = a^3

Longest Diagonal of cube = $a\sqrt{3}$

3) Volume related units

1 liter = 1000 cubic cm.

1000 liters = 1 cubic meter = 1 kiloliter

1 cubic cm = 1000 cubic mm

1 cubic meter = 1000000 cubic cm.

Chapter - 10

statistics

Dear students! we learn about the organisation of data, the method of measuring central tendencies in our in previous classes. Statistics, a branch of mathematics, has been used in India since ancient times, which is evident from the following example.

For example, in the Mahabharata period, during Damayanti's Swayamvar king Nala was asked to guess the number of leaves and fruits on a tree based on king Ritupurnas result.

In our daily lives we find data being analysed in in newspapers electronic media, magazines and other means of communication about the Factual and comparative information on rainfall status, agricultural production etc. We keep using data in some form or the other throughout our life, hence it becomes very important for us to derive meaningful information from these data as per our wish. The study related to interpreting meaningful information is done in this branch of mathematics called **statistics**.

We represent the data in the form of pictures and graphs, to study the mean, median and mode in detail.

Data Collection:

On the basis of collection of data, it can be divided into two parts, namely.

1) Primary data
2) Secondary data

1) **Primary data:**

Data is collected directly for the first time Then they are called **primary data. as**- weights and heights, of Veda Bhushan fourth year students in Gurukul.

2) Secondary data :

Data taken frome already published or unpublished, (data is collected by many institutions but is not published, such materials can be obtained from files, registers, documents etc) is called **secondary data.**

Presentation of Data –

After collecting the data, its presentation should be meaningful and easily understood by all. Therefore, the data is also divided into two parts on the basis of presentation.

1) Unclassified data   2) Classified data

1) Unclassified Data:

The presentation of data compiled (collected), is presented in the form in which it was collected, is called **unclassified data** (raw).

For example: The scores of 10 students of Veda Bhushan IV are as follows.

9 , 7 ,8 , 6 , 5 , 7 , 7 , 8 , 9 , 8

2) **Classified Data :**

It is necessary to present the data in an organised way based on deffernt titles for further studies. Such gruoped data is called classified data.

Frequency:

In a data set, the number of times for which a perticular observation is repeated, is called **frequency.** It is represented by the latter 'f'.

Observation: Each value of the collected data is called **an observation.**

For example: 2 , 4, 7, 7, 6, 9, 1, 5 The number of observations in is 8.

Do and Learn –

What is the number of observations in 8 , 4, 7, 1 and 3 ? (.........)

Range:

In a data set, the difference between the maximum and minimum values of observations is called its range.

Range = Maximum value – minimum value

Example: Find the range of the following data set.

6 , 12 , 21 , 25 , 91 , 57

Solution : Minimum value = 6, Maximum value = 91

Therefore, **range = maximum value – minimum value**

= 91 – 6

= 85

Frequency table:

When the collected data is first arranged in ascending or descending order and presented in the form of a table is called **frequency table.** Let us recall some essential definitions related to Frequency table for our ferther studies.

Tele marks :

While organising the observation of a data set in the form of a frequency table, a verticle line is drawn against the respective observation if an observation is reapited for five times, the fifth line is drawn to cross the first four lines all ready drawn. This is repeated for ferther observations, this makes easier for us to present the deta.

For example : The collection of 15 data in ascending order is given below.

7 , 7, 7, 8, 8, 8, 8, 8, 9, 9, 9, 9, 9, 9, 9

Their matching tele marks will be as follows.

A table is prepared from the above data in the following manner.

1) In the first column write the uniqe observations without leaving any of them.
2) In the second column, draw tele marks (vertical lines) for each observation of the data set.
3) In the third column, the frequency of each observation is calculated and written.

Number	tally (tel) sign	frequency
7	׀׀׀	3
8	~~׀׀׀׀~~	5
9	~~׀׀׀׀~~ ׀׀	7
		$\sum f = 15$

In this way the frequency table can be prepared as a representation of unclassified data. If the number of data is very large, the data set is simplified by placing them in groups.These groups are called **classes and their deffrence is called class interval. The lowest number of each class is called the lower class limit,** and the maximum number is called **Upper Class Limit.**

Let us learn to present data in the form of a table by an example.

Example: In a school, the number of students of all four classes is 30. Each student was asked plant some saplings around the school. Number of saplings planted By each student is given below.

15 , 17, 18, 12, 34, 34, 35, 43, 18, 29, 23, 40, 35, 25,

26, 28, 13, 10, 3, 4, 3, 0, 14, 25, 25, 0, 2, 47, 19, 20

Solution:

number of plants	tally mark	frequency
0 – 10	~~IIII~~ I	6
10 – 20	~~IIII~~ IIII	9
20 – 30	~~IIII~~ III	8
30 – 40	IIII	4
40 – 50	II	3
		$\sum f = 30$

This method of presenting data is called **grouped frequency table.** We can easily make inferences and draw conclusions. Based on the simplified table.

Exercise 10.1

1. Select the correct option from the following multiple choice questions-

(a) The frequency of the tally mark ~~llll~~ ~~llll~~ lll is-

(I) 5 (II) 10 (III) 13 (IV) 15

(b) The telemark of frequency 13 is-

(I) lllllllllllll (II) ~~llll~~ llllllll (III) ~~llll~~ ~~llll~~ l (IV) ~~llll~~ ~~llll~~ lll

(c) The frequency of 17 in 17,15,19,20,17,5,6,9,13,18,17,5,9,6,19, is-

(I) 3 (II) 5 (III) 15 (IV) 20

(d) The frequency of 5 in 9,7,3,5,11,3,13,5,6,3,9,10,5,9,7,5 is-

(I) 1 (II) 2 (III) 3 (IV) 4

(e) The upper limit of class interval 15-25 is-

(I) 15 (II) 25 (III) 40 (IV) 20

(f) The lower limit of class interval 31-35 is-

(I) 4 (II) 31 (III) 66 (IV) 35

(g) What is the difference between the maximum and minimum values of the data called-

(I) Range (II) lower limit

(III) oo boundary (IV) Frequency

(h) The range of figures 39 , 25, 15, 13, 32, 16 is-

(I) 32 (II) 26 (III) 25 (IV) 15

(i) The range for figures 11,32,51,49,49,6,14 are-

(I) 6 (II) 45 (III) 49 (IV) 51

2. What do you understand by primary data and secondary data ?
3. What do you understand by collection of data ?
4. What do you understand by presentation of data, and into how many parts is it divided?
5. What is frequency ?
6. Explain the term observation.
7. Fill in the blanks: Range = ... − ....
8. Following is the weight in kilograms of 20 students of Ved Bhushan fourth year.

   17 , 20, 32, 30, 25, 27, 28, 29, 18, 21, 23, 23, 24, 25, 25, 28, 18, 17, 30 , 25 Write the above data in tabular form.
9. Before admission in Veda Pathshala, a test was conducted. The maximum marks of the question papers is 20. The scores of 10 students are given below: 15 , 15, 15, 17, 17, 18, 18, 19, 17, 19

   Present the above data in form of a table.
10. Construct a frequency table for the following data by taking the class intervals of 10.

    13 , 11, 8, 19, 0, 44, 27, 10, 8, 35, 13, 27, 31, 18, 42, 23,

    19, 34, 19, 27, 43, 17, 7

Graphical Representation of statistical data –

From the previous Exercise, we have learned how the given data can be tabulated. by presenting the data pictorially, in such a way that it not only looks good but also provides convenience in study. Let us study about bar graphs.

1) **Bar Graph:**

Through this, statistical data related to any one entity is displayed through bar graphs.

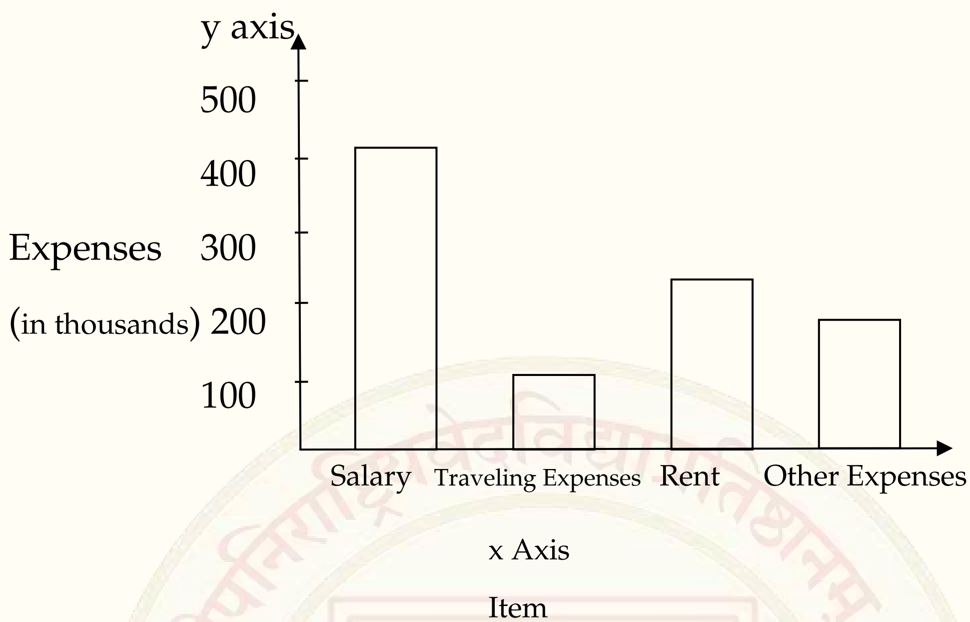
In this graphical representation, bars of equal width are drawn on the 'x' axis and height parallel to the 'y' axis as per the given data.

Let us learn to draw bar graphs by examples.

Example: The expenditure incurred on various items by a business establishment are as follows: Show the data using a bar graph.

Item	Expenses (in thousands)
Salary	400
traveling expenses	100
Rent	250
other expenses	200

Solution: In a bar graph, the width of each bar and the distance between the bars on the 'x' axis make the picture clear and understandable.



Do and Learn – The number of mantra memorized by five Vedik students in the month of April 2020 is as follows, draw a bar graph to show the data.

Vedapathi Vidya Rathi	Number of months quarantined
Diwakar	200
Avinash	150
Ashish	250
Gautam	300
Saket	100

Histogram:

Histogram is a rectangular representation of classified and continuous frequency data, in which there are class intervals and the height of the rectangles is according to the frequency of those classes.

In this the class interval is measured by taking appropriate scale 'x'. On the axis and the frequency is marked on the 'y' axis by taking appropriate scale. Let us learn to draw a histogram by an example.

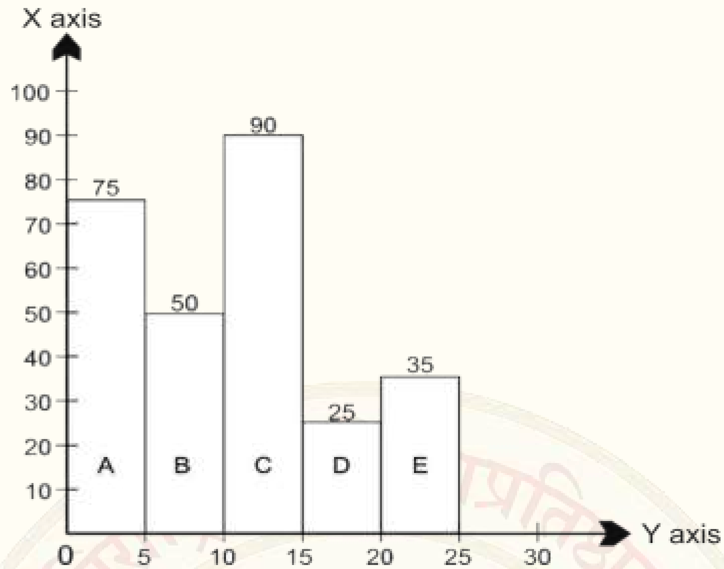
Example: The number of students of different ages in a school is as follows. Make a histogram for the following frequency table.

years (age in years)	0 – 5	5 – 10	10 – 15	15 – 20	20 - 25
number of students	75	50	9 0	25	35

In this the frequency table is classified, continuous and the class interval are equal, hence, the Class interval which means age in years will be marked on the 'x' -axis.

Solution: Since the number of students in the class interval (0 – 5) is 75, on a graph sheet, x - axis is drawn and the class intervals of ages are marked.

On y– axis, frequency is marked. A rectangle bar as drawn above the class itervals of heights as per the frequency. For Example for 0 – 5, rectangular bar of height 75 is drawn. The process is repeated for all they entries.

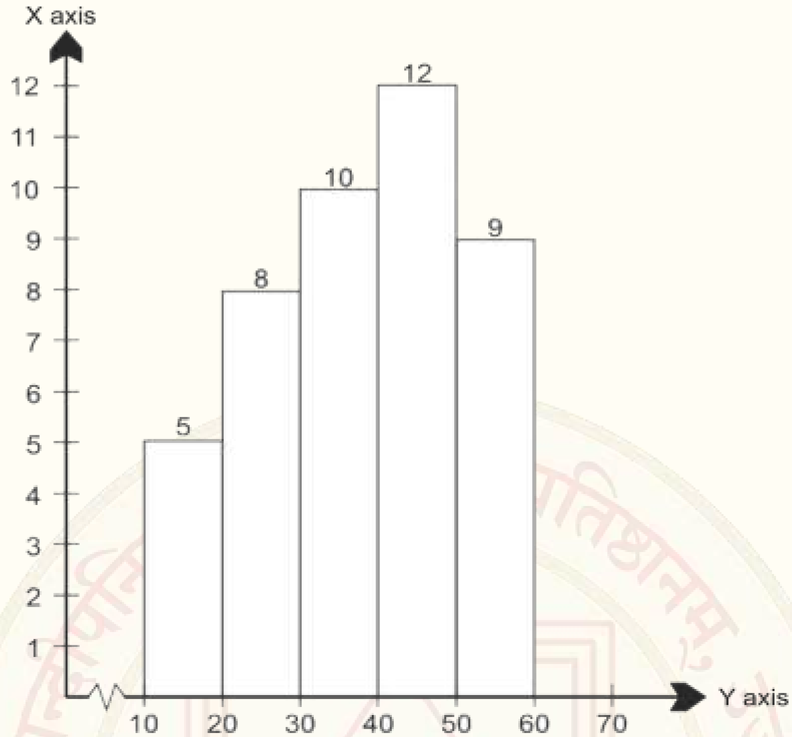


Hence it is clear that in all these rectangles 1 cm. And height is equal to frequency, thus, the area of rectangles will be proportional to frequency.

Example: Draw a histogram for the following data.

Class Interval	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency	5	8	10	12	9

Solution : In this, the class interval is 10. Hence, takeing the scale on 'x' axis as 10 units = 1 cm. And on 'y' axis 1 unit = 0.5 cm, the following histogram can be drawn



Remember: - Kink $\boxed{-\!\wedge\!\!\vee\!\!-}$ symbol is used when the class interval does not start from zero.

Exercise 10.2

1. A family with an income of Rs 20,000 had planned its monthly expenditure under various heads.

Item	Expenditure
groceries	8
Rent	4
Education	5
medicines	2
Entertainment	3

Make a bar graph of the above data.

2. Out of 40 students of a Veda Pathshala, the number of students appearing in the merit list of Veda examination is as follows.

Veda	Batuk coming in merit list
Rigveda	8
yajurveda	8
Samveda	7
Atharvaveda	6

Make a bar graph of the above data.

3. A Rajdhani Express train stops at the following number of stations in different states.

State	number of train stops
M.P.	5
Gujarat	3
Uttar Pradesh	7
Uttarakhand	2

Make a bar graph of the above data.

4. Make a histogram of the following frequency table.

class Interval	10 – 20	20 – 30	30 – 40	40 – 50
frequency	5	10	20	15

5. Make a histogram for the following data.

score	0 - 10	10 – 20	20 – 30	30 – 40
No.of student	5	4	8	10

6. Draw histogram for the following frequency table.

Class Interval	0 - 5	5 – 10	10 – 15	15 – 20	20 – 25
frequency	15	10	5	20	25

We learned -

1) There are mainly two types of data.

   a) Primary data   b) Secondary data

2) The following is the method of collecting primary data.

   a) personal investigation   b) indirectly

3) Following are the methods of collecting indirect data.

   a) Through schedules and question papers

   b) From receiving information from correspondents

4) Presentation of data is divided into two parts.

   a) Ungrouped data (**for example**: 8 , 7, 5, 10, 2, 3 ,……)

   b) Grouped data (**for example**: 0-10 , 10-20 , ………..)

5) **Range** – The difference between the maximum and minimum values of observations is called the **range .**

$$\textbf{range} = \textbf{maximum value} - \textbf{minimum value}$$

6) The lowest and highest values of any class (**for example:** 0 – 10, 10– 20, 20-30) are called the lower and upper limits of that class respectively.

7) **Frequency –** The number of times the data is repeated is called the frequency of the data, it is denoted by 'f'.

8) Various types of data can be presented graphically through graphs and histograms.

Chapter 1 1

Probability

My dear Students ! Till now we have solved questions, which has a definite answer, **such as:**

1) $2 + 2 = 4$

2) 3% of $100 = 100 \times \frac{3}{100} = 3$

3) $15 \div 3 = 5$

Calculating the the circumference ($2\pi r$) and area (πr^2) of a circle of radius r.

There are some questions which do not have a definite answer.

1) It may rain today.

2) Perhaps the prices of diesel and petrol will increase.

3) Maybe they will be on the way.

Of the students of a school-

4) The studants of a ved pathshala winnintg a competition on antakshri.

5) There are full chances of his success.

6) The chances of India winning the toss in today's match are 50-50.

Here we cannot expect a definite answer from all the above mentioned statements. There remains a feeling of doubt, 'probability' and coincidence in the answer to all the statements.

For example -

1) “It will probably rain today ” would mean that it may or may not rain.

Thus, the answers to the above statements can be 'yes' as well as 'no'. With the help of 'probability', we can numerically calculate the 'possiblities’ of the occurrence of events. Probability of an event conceptual or numerical representation of chances of occurence of the event. In other words, the calculation of uncertainty is called **probability**. It is the measure of posible occurence of the event.

Probability is the numerical value of the occurrence of an event, for an event E, the probability of occurrence can be found in the following way.

$$P\ (E) = \frac{\text{number of favovrable outcomes}}{\text{total number of outcomes}}$$

$$P\ (E) = \frac{n\ (E)}{n\ (S)}$$

Activity 1 : To understand probability, we will do the following activity. Take a coin and toss it for 10 times. Count the number of times that heads and tails comeup on tossing, and write them in the following table.

number of times coin tossed	number of heads coming up	number of tails coming up
10		

Write the values of the fractions given below.

$$P(E_1) = \frac{\text{number of times heads coming up}}{\text{total number of times coin toss}}$$

$$P(E_2) = \frac{\text{number of times tails coming up}}{\text{total number of times coin toss}}$$

On tossing a coin, the outcomes can be either a head or tail. In the above, P (E_1) and P (E_2) are **the probabilities** of getting heads and tails respectively. In this , The sum of the values of P (E_1) and P (E_2) is 1.implies

$$P(E_1) + P(E_2) = 1$$

Remember : The sum of the probabilities of the total events obtained in an experiment is always 1.

Experiment (random experiment):

The collection of possible outcomes of a event such that the occurrence of every event is certain and cannot be predicted exactly before is occurrence is called an experiment.

Result:

The outcome obtained after conducting an experiment once is called **result.** For example, in the experiment of tossing a coin there are two possible outcomes: heads or tails.

Event:

One or more outcomes of any experiment is called **an event.**

Example : Getting an even number by throwing a dice.

Sample Space : The set of all possible outcomes of an experiment is called the sample space of that experiment.

The sample space and sample points (event) of the experiment are explained in the table below.

random experiment	sample space	sample point(event)
tossing a coin	S= {H,T}	H,T
tossing a die	S ={1,2,3,4,5,6}	1 ,2 ,3 ,4 ,5 ,6
tossing two coins together	S={(H,H),(TT), (H, T) ,(T, H)}	(H , H),(T , T), (H , T), (T , H)

Some important findings of Probability:

1) The sum of all the probabilities of all possible outcomes is 1.

2) P(occurrence of the event) + P (non-occurrence of the event) = 1

Or $P\ (E)\ +\ P\ (\bar{E})\ =\ 1$

3) The probability of an event is always between 0 and 1.

$$0 \leq P(E) \leq 1$$

4) The probability of an impossible event is 0 and the probability of a certain event is 1.

Probability is the numerical value of the occurrence of an event. This can be determined as follows.

$$\boldsymbol{P(E)} = \frac{\text{number of favovrable outcomes}}{\text{total number of outcomes}}$$

$$\boldsymbol{P(E)} = \frac{\boldsymbol{n(E)}}{\boldsymbol{n(S)}}$$

Example : Find the probability of getting heads when a coin is tossed once.

Solution : Sample space S = (H, T)

Total number of results $n(S)$= 2

Event of coming to favourable event (A) = H

Number of favorable outcomes /total number of outcomes n (A) = 1

$$\textbf{probability} \boldsymbol{P(A)} = \frac{\text{number of favovrable outcomes}}{\text{total number of outcomes}} \frac{\text{n(A)}}{\text{n(S)}}$$

$$= \frac{1}{2}$$

Example : Find the probability of getting a number less than 3 on throwing a simple dice.

Solution : Sample space S = {1,2,3,4,5 , 6}

Total number $n(s)$ of results = 6

Event (E) of getting an outcomes less than 3 = {1,2}

Therefore , number of outcomes favorable to the event $n(E)$= 2

Now ,

$$\textbf{Probability}\, P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{2}{6} \text{ or } \frac{1}{3}$$

Hence the probability of getting a number less than 3 is $\frac{1}{3}$

Example : Find the probability of getting an odd number on top when a cubic die is thrown?

Solution : Sample space (S) = {1,2,3,4,5,6}

Total number of results n(s) = 6

Occurrence of odd digit (E) = {1,3,5}

Therefore , the number of outcomes favorable to event E is n(E) = 3.

Probability $P(E)$ of getting an odd number = $\frac{n(E)}{n(S)}$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

Hence, there $\frac{1}{2}$ is a probability of getting an odd number on the top of the die.

Exercisen11.1

1. Select the correct option from the following multiple choice questions.

(a) What is the value of probability of a certain event ?

(I) 0 (II) $\frac{1}{2}$ (III) 1 (IV) $\frac{3}{2}$

(b) If the probability of an event is denoted by P (E) then -

(I) $P(E) \leq 0$ (II) $P(E) \leq 1$ (III) $0 \leq P(E) \leq 1$ (IV) $-1 \leq P(E) \leq 1$

(c) Which of the following numbers cannot be the probability of an event ?

(I) $\frac{1}{2}$ (II) $\frac{-1}{2}$ (III) $\frac{1}{4}$ (IV) 1

(d) If a coin is tossed once, the probability of getting a head is –

(I) $\frac{1}{2}$ (II) $\frac{1}{3}$ (III) $\frac{1}{4}$ (IV) $\frac{1}{6}$

(e) If a dice is thrown once , the probability of getting an even number is -

(I) $\frac{1}{2}$ (II) $\frac{1}{3}$ (III) $\frac{1}{4}$ (IV) $\frac{1}{6}$

2. Write the following definition –

a) probability b) sample space

3. Find the probability of getting a tail when a coin is tossed.

4. Find the probability of getting more than 4 on top when a dice is thrown.

5. If a coin is tossed 10 times, heads comes up for7 times. Then find the probability of getting a head.

6. Find the probability of getting an even number when a die is thrown.

7. Find the probability of getting 0 when a die is thrown.

8. If a dice is thrown twice , the probability of getting an odd number is-

9. Fill in the blanks.

   a) The probability of an impossible event is……………………..

   b) The value of probability is always between…………..

   and…………...

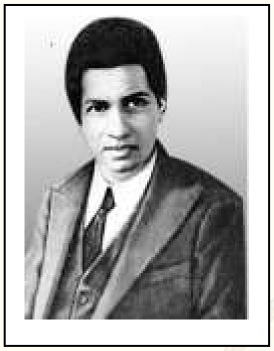
   c) The probability of a certain event is…………..

We learned -

1) An event of an experiment is a collection of some results of the experiment.
2) The experimental probability of an event E is $P(E)$.
3) Probability is a numerical representation of the chances of an event occurring.
4) probability $P(A) = \frac{\text{number of favovrable outcomes}}{\text{total number of outcomes}} \frac{n(A)}{n(S)}$
5) The sum of probabilities of all possible outcomes of an experiment is 1.
6) The probability of an event is always between 0 and 1.

Introduction and contribution of Indian mathematician

- **Srinivas Ramanujan (1887 – 1920)-**



The great mathematician Srinivas Ramanujan was born on 22 December 1887. He was born in a small village called 'Erode' near Kumbha Konam in Tamil Nadu state. Ramanujan was born in a poor Brahmin family. **His father used to recite Vedas in the temple for livelihood.** Along with this, he was maintaining accounts of a shop. Ramanujan from the beginning He had an inquisitive nature and a sharp mind. He had a special interest in mathematics and till high school he always stood first in his class.

After passing the high school examination, Ramanujan took admission in Kumbhakonam College, he had special interest in mathematics. On the basis of his unique ability in mathematics, he taught mathematics to college students and his research work in mathematics also continued. His entire treasure was two handwritten notebooks. He used to tell his friends that if he died, These booklets should be given to Professor Singar Velu or Professor Edward Ross.

In 1913, he wrote a letter to professor G. H. Hardy of Cambridge University. Along with that letter, he sent 120 theorems. Professor Hardy was greatly inmpressed by Ramanujan's work, and called him to England in 1914.

He reached London on 14th April 1914, on the invitation of professor Godfrey Harold Hardy. As soon as he reached England, Started working hard on his field of research. Ramanujan and Hardy worked continuously to punlished 9 research papers in 1915. In all Ramanujan published 21 research papers in collaboration with professor Hardy. He was elected as a fellow of London Mathematical Society on 6th December 1917. On the basis of his extraordinary talent , Ramanujan was Elected as a Fellow of the Royal Society on May, 1918.

On that year 104 scholars were Nominated for fellowship, Ramanujan was one among the 15 scholars. He was the first Indian to be selected for this honour. He was the first Indian citizen to accomplish so many achievements at such a young age and he spread the word of his discovery in other countries including India.

He was appointed as a professor by Madras University. On 27 March 1919, he reached India after his visit to England. He was given a grand welcome on his return. Following are the important works of Ramanujan.

1) Whole numbers 2) Infinite series 3) Continuous fraction 4) Series 5) Combined numbers etc. Ramanujan was a an ardent practioner of Indian traditions. While going to England, he had promised his father that "he will live like an Indiam even in England,"and will not do anything that will hurt Indianness".

He kept his promisc fully.While being an abroad, in addition to his research work, he did all his works by himself. He used to discuss mathematical problems with others for hours together. His studiousness was exemplary. Till his last moment of this life he was completely focussed on his studies, research and documentation of work without bothering his shortcomings.

Ramanujan was a brilliant mathematician. Even during his illness, he used to solve mathematical hypotheses while lying on his bed.

Once he was admitted in a hospital, Prof. Hardy had came to visit him. The number of the taxi in which Hardy had come. Was 1729 (7 ×13 ×19). Pro. Hardy considered this number inauspicious. Hearing this, Ramanujan said - This is the smallest number , which we can express in two ways by the sum of two cube numbers -

$$1729 = 12^3 + 1^3 \text{ and } 10^3 + 9^3$$

On April 26,1920, Srinivas Ramanujan passed away at the age of 33 years due to the illness of T.B. The whole of India including other countries where he had become popular mourned the loss of this great Indian mathematician.The number 1729 is known as Ramanujan number.The auspicious occasion of the day of birth of this great mathematician of India, is celebrated as IT day and National Mathematics

A film has been released in the year 2015 on the entire life of srinivas Ramanujan and its name is “Man who knew infinity”.

❖ **Krishna Venkateshwar Sharma (1919-2005)**

Krishna Venkateshwar Sharma ji Did M. A. in Sanskrit from Trivandrum. and later joined V.Raghavan of Madras University to work on Catalog Catalogorum. Soon he started working on Kerala’s astronomy.

He edited many important books. Example Swaroop Hari Dutt's Grahakarani Bandhan , Neelkanth's Siddhant Darpan, Madhav's Gola Dipika etc. During this period, Sharma worked with the renowned scholar T.S.Kuppana Sastri for an edition of Vakyakaran (1961). During 1975–80, Sharma was transferred as Director of the Vishveshwar Nand Institute at Hoshiyarpur. During this period of service, he published more than 50 books including many important works such as History of Hindu Astronomy of the Kerala School, Bhaskaracharya's Lilavati, Shankara's Kriya Krama Kari, Neelkanth's Tantra Collection, Shankara's Yukti. Deepak, Neelkanth Of Astronomy etc. Sharma returned to Madras in 1983 And his work continued actively until his death in 2005. His important publications during this period are: 'Indian Astronomy: A Source Book' (1985) with B.V. Subba

Rayappa, based on Varahamihir's Pancha Siddhantika a book with T.A. Sakuppana Sastri in (1993); And his biggest work is the translation and edition of Yuktibhasha, published in 2008 K. Ram Subramaniam, Published articles with M.S. shri Ram and M. D.

❖ **Shakuntala Devi** (1929-2013)

Shakuntala Devi (4 November 1929 – 21 April 2013), whom we know as the "Human Computer", was blessed with amazing talent since childhood. Seeing her talent, her name was included in the 'Guinness Book of World Records' in 1982.

Shakuntala Devi was born in an orthodox Kannada Brahmin family in Bengaluru, the capital of Karnataka. Shakuntala Devi's father used to perform tricks in the circus. When she was playing cards with her father at the age of 3, her father discovered that his daughter had the capability to solve mental ability questions.

Shakuntala demonstrated her abilities on calculations in a major event at the University of Mysore at the age of 6 years. In the year 1977, Shakuntala had told the 23^{rd} square root of the number 201 without pen and paper. She had given the product of two 13 digit numbers in 26 seconds.

आदर्श प्रश्नपत्र/ Model Que. Paper : IV / गणित /
वेदभूषण चतुर्थ-वर्ष / Vedabhushan Fourth Year/
कक्षा 9वीं / पूर्व मध्यमा - I / Class 9th / Purv Madhyama - I
विषय - गणित

पूर्णांक – 100 **समय – 3 घण्टे**

• सभी प्रश्न हल करना अनिवार्य हैं। • सभी प्रश्न के उत्तर पेपर में यथास्थान पर ही लिखें। • उत्तीर्णता हेतु न्यूनतम 40% अंक निर्धारित हैं। • आदर्श प्रश्न पत्र का छात्रों को लिखित परीक्षा हेतु अभ्यास कराएँ।	• It is mandatory to attempt all the questions. • Write down the answers at the appropriate places provided. • The minimum pass marks are 40%. • The model question paper should be used by the students for written examination practice.

प्रश्न- 01 सही विकल्प के सामने (✓) चिह्न लगाइए – $(\mathbf{10 \times 2 = 20})$

Question - 01. Tick (✓) the correct option.

1. निम्नलिखित में से कौन सा कथन सत्य नहीं है –

   Which of the following statement is not true -

   (अ) किन्हीं दो परिमेय संख्याओं का गुणनफल करने परिमेय संख्या प्राप्त होती है।
   Multiplication of any two rational numbers gives a rational number.

   (आ) किन्हीं दो परिमेय संख्याओं के बीच अनंत परिमेय संख्या होती है ।
   There are infinite numbers of rational numbers between any two rational numbers.

   (इ) $\sqrt{2}$ परिमेय संख्या है ।
   $\sqrt{2}$ is a rational number.

   (ई) दो परिमेय संख्याओं को जोड़ने पर परिमेय संख्या प्राप्त होती है ।
   Adding of two rational numbers gives a rational number.

2. प्रायिकता का मान निम्न में से किनके मध्य होता है –

The value of probability lies between which of the following -

(अ) 0 और 1 के मध्य

Between 0 and 1

(आ) 2 और 3 के मध्य

Between 2 and 3

(इ) 3 और 4 के मध्य

Between 3 and 4

(ई) इनमे से कोई नहीं

None of these

3. बहुपद के सन्दर्भ कौन-सा कथन असत्य है -

Which statement is false regarding the polynomial -

(अ) बहुपद $\frac{\pi}{2}x^3 + x$ में x^3 का गुणांक $\frac{\pi}{2}$ है।

In the polynomial $\frac{\pi}{2}x^3 + x$ of the $\frac{\pi}{2}$

(आ) बहुपद में चरों की घात पूर्ण संख्या होती है ।

The degree of a polynomial is a whole number of variables.

(इ) तीन घात वाले बहुपद त्रिघात बहुपद कहलाते हैं ।

Polynomials of degree three are called cubic polynomials.

(ई) $x^{-5} + y$ एक बहुपद है।

$x^{-5} + y$ is a polynomial.

4. वैदिक गणित के सन्दर्भ में कौन -सा कथन असत्य है –

Which statement is false in the regarding of Vedic Mathematics -

(अ) बीजांक का अर्थ है- 'आङ्किक योग'

The meaning of 'Bijank' is 'numerical additions'.

(आ) आधार संख्या से कम या अधिक संख्या को विचलन कहते है ।

A number less or more than the base number is called deviation.

(इ) आधार संख्या हमेशा 10 के गुणज के रूप में होती होती है ।

The base number is always in the form of multiples of 10.

(ई) 102 में आधार 100 एवं विचलन (–2) है।

In 102 has base 100 and deviation (–2)

5. वृत्त के सन्दर्भ में कौन-सा सत्य नहीं है -

Which of the following statement is not true with respect to circle?

(i) वृत की परिधि पर किन्हीं दो बिंदुओं को मिलाने वाली रेखाखण्ड जीवा कहलाती है।

A line segment joining any two points on the circumference of a circle is called a chord.

(ii) वृत्त की व्यास सबसे बड़ी जीवा होती है ।

The diameter of the circle is the largest chord.

(iii) वृत्त के व्यास को आधा करने पर वृत्त की त्रिज्या प्राप्त होती है ।

Radius of the circle is obtained by halving the diameter of the circle.

(iv) यदि वृत्त का व्यास 14 सेंटीमीटर है, तो उसकी त्रिज्या 28 सेंटीमीटर होगी ।

If the diameter of the circle is 14cm, then its radius will be 28cm.

(अ) केवल (i)

Only (i)

(आ) केवल (i) एवं (iv)

(i) and (iv) only

(इ) दोनों (i) एवं (iii)

Both of (i) and (iii)

(ई) केवल (iv)

Only (iv)

6. त्रिभुज के सन्दर्भ में कौन-सा सत्य नहीं है-

Which is not true in the regarding of triangle -

(अ) त्रिभुज में तीन भुजाएँ एवं तीन कोण होते हैं ।

A triangle has three sides and three angles.

(आ) त्रिभुज के तीनों कोणों का योग 90^0 होता है।

The sum of all the three angles of a triangle is 90^o

(इ) त्रिभुज कि किसी दो भुजा का योग, अन्य भुजा से सदैव अधिक होता है ।

The sum of any two sides of a triangle is always greater than the other side.

(ई) वह त्रिभुज जिसका एक कोण समकोण (90^0) हो समकोण त्रिभुज कहलाता है ।

A triangle whose angle is a right angle (90^0) is called a right angle triangle.

7. समकोण त्रिभुज के सन्दर्भ में कौन-सा कथन सत्य नहीं है ।

Which statement is not true with respect to a right angle triangle?

(अ) समकोण त्रिभुज के न्यूनकोण के संलग्न क्षैतिज भुजा आधार कहलाती है।

The horizontal adjacent side to the acute angle of a right triangle is called base.

(आ) समकोण त्रिभुज के न्युनकोण के संलग्न ऊर्ध्वाधर भुजा लम्ब कहलाती है ।

The vertical adjacent side to the acute angle of a right-angled triangle is called perpendicular.

(इ) समकोण त्रिभुज की सबसे लम्बी भुजा कर्ण होती है ।

The longest side of a right triangle is the hypotenuse.

(ई) समकोण त्रिभुज की दो भुजाओं का योग, तीसरी भुजा से कम होता है ।

The sum of two sides of a right-angled triangle is less than the third side.

8. निम्न में कौन-सा सूत्र सत्य नहीं है –

Which of the following formula is not true -

(अ) त्रिभुज का क्षेत्रफल = $\frac{1}{2}$ × आधार × ऊँचाई

Area of triangle = $\frac{1}{2}$ × base × height

(आ) त्रिभुज का अर्द्धपरिमाप = $\frac{\text{त्रिभुज का परिमाप}}{2}$

Semi-perimeter of triangle = $\frac{\text{Perimeter of triangle}}{2}$

(इ) घनाभ का विकर्ण = $a\sqrt{3}$ इकाई

Diagonal of cuboid = $a\sqrt{3}$ units

(ई) घनाभ का आयतन = लम्बाई × चौड़ाई × ऊँचाई

Volume of cuboid = length × breadth × height

9. कथन (A) – घन का आयतन = a^3

Assertion (A) - Volume of cube = a^3

कथन (R) – यदि घन की एक भुजा 5 मीटर है, तो घन का आयतन 125 घन मीटर होगा।

Statement (R) - If one side of a cube is 5 meters, then the volume of the cube will be 125 cubic meters.

(अ) A एवं R दोनों सही है। R, A की सही व्याख्या करता है।

Both A and R are correct. R is the correct explanation of A.

(आ) A एवं R दोनों सही है। R, A की सही व्याख्या नही करता है।

both A and R are correct. R does not explain A correctly.

(इ) A सही है परन्तु R गलत है।

A is correct but R is incorrect.

(ई) A गलत है परन्तु R सही है।

A is wrong but R is correct.

10. निम्न में कौन-सा कथन सही नहीं है -

Which of the following statement is not correct -

(अ) अधिकतम मान एवं न्युनतम मान के अन्तर को परिसर कहते है ।

The difference between the maximum value and the minimum value is called range.

(आ) आंकड़ों की जितनी बार पुनरावृत्ति होती है उसे आंकड़ों की आवृत्ति कहते हैं।

The number of times the data is repeated is called the frequency of the data.

(इ) प्रत्यय व्यक्तिगत अन्वेषण आवृत्ति प्राथमिक आंकड़ों के सङ्कलन करने की विधि है।

Suffix personal Inquiry Frequency is the method of collection of primary data.

(ई) अधिकतम मान एवं न्युनतम मान के अन्तर को आवृत्ति कहते है ।

The difference between the maximum value and the minimum value is called frequency.

प्रश्न - 02. रिक्त स्थानों की पूर्ति कीजिए – **5 × 2 = 10**

Question - 02. Fill in the blanks

1. π एक .... संख्या है ।

   π the number is ....

2. संख्या 89 का विचलन .... है ।

   Deviation of number 89 ....

3. वृत्त का व्यास, त्रिज्या की माप का .... गुणा होता है ।

Of a circle is .... times the measure of the radius.

4. पाइथागोरस प्रमेय केवल .... त्रिभुज पर लागु किया जा सकता है ।

Pythagoras theorem only can be applied to triangles ....

5. घन का सम्पुर्ण पृष्ठीय क्षेत्रफल .... है ।

Total surface area of the cube is ....

प्रश्न - 03. निम्नलिखित युग्मों के मिलान पर विचार कीजिए – **5 × 2 = 10**

Question - 03. Consider matching the following pairs -

1.	विनकुलम् संख्या Vinkulm Number	क.	1
2.	निश्चित घटना की प्रायिकता Probability of certain event	ख.	ऋणात्मक संख्या Negative number
3.	सर्वांगसम चिह्न Congruent sign	ग.	$\sum$
4.	सिग्मा चिह्न Sigma sign	घ.	$\cong$
5.	असम्भव घटना की प्रायिकता Probability of an impossible event	ङ.	0

उपर्युक्त युग्मों के आधार पर सही विकल्प का चयन कीजिए –

Select the correct option based on the above pairs -

(अ) (1) (ख), (2) (क), (3) (ग), (4) (घ), (5) (ङ)

(आ) (1) (ख), (2) (क), (3) (ग), (4) (ङ), (5) (घ)

(इ) (1) (ङ), (2) (ग), (3) (घ), (4) (क), (5) (ख)

(ई) (1) (ख), (2) (क), (3) (घ), (4) (ग), (5) (ङ)

प्रश्न- 04 सत्य / असत्य कथन पर विचार कीजिए – **$5 \times 1 = 5$**

Question - 04. Consider the true / false statement -

1. जिस बीजीय व्यंजक में चर की घातांक पूर्ण संख्या हो, बहुपद कहलाता है ।
   An algebraic expression in which the exponent of the variable is a whole number is called a polynomial.
2. संख्या 531 का बीजांक 9 है ।
   The bijank number of 531 is 9.
3. वृत्त के आन्तरिक भाग को परिसीमा कहते हैं ।
   The internal part of the circle is called the boundary.
4. ठोस आकृतियों द्वारा घेरा गया स्थान (क्षेत्र) आयतन कहलाता है ।
   The space (region) enclosed by solid figures is called volume.
5. (कर्ण)^2 + (लम्ब)^2 = (आधार)^2
   (Hypotenuse)^2 + (Perpendicular)^2 = (Base)^2

उपर्युक्त कथनों को पढ़कर सही विकल्प का चयन कीजिए –

Read the above statements and choose the correct option -

(अ) (1) सत्य, (2) असत्य, (3) सत्य, (4) सत्य, (5) सत्य
(1) True, (2) False, (3) True, (4) True, (5) True

(आ) (1) सत्य, (2) असत्य, (3) सत्य, (4) असत्य, (5) सत्य
(1) True, (2) False, (3) True, (4) False, (5) True

(इ) (1) सत्य, (2) सत्य, (3) असत्य, (4) असत्य, (5) असत्य
(1) True, (2) True, (3) False, (4) False, (5) False

(ई) (1) असत्य, (2) सत्य, (3) असत्य, (4) असत्य, (5) सत्य
(1) False, (2) True, (3) False, (4) False, (5) True

प्रश्न - 05. अति लघूत्तरीय प्रश्न – **$10 \times 2 = 20$**

Question -05. Very Short Question -

1. $\frac{1}{2+\sqrt{3}}$ के हर का परिमेयीकरण कीजिए। / Rationalize the denominator of $\frac{1}{2+\sqrt{3}}$

----

2. वैदिक गणित में कितने सूत्र एवं कितने उपसूत्र है।
   How many sutras and sub-sutras are there in Vedic Mathematics?

----

3. वैदिक गणित के कोई पाँच सूत्र अर्थ सहित लिखिए ।

Write any five sutras of Vedic Mathematics with meaning.

----

4. सर्वांगसमता की अवधारणा लिखिए ।

Write the concept of congruence.

----

5. बहुपद ($3x^2 + 4x + x$) में x से भाग दीजिए ।

Divide the polynomial ($3x^2 + 4x + x$) by x

6. त्रिभुज की सर्वांगसमता के कितने नियम है नाम लिखिए।

Write the rules of congruence of triangles.

7. एक घन की कोर 10 मी. है तो घन का आयतन ज्ञात कीजिए।

The edge of a cube is 10 m. find the volume of cube.

----

8. एक सिक्के के उछालने पर पट आने की प्रायिकता ज्ञात कीजिए।

Find the probability of getting tail when a coin is tossed.

----

9. यदि किसी समकोण त्रिभुज की सबसे बड़ी भुजा कौन-सी है।

What is the largest side of a right angled triangle?

----

10. एक त्रिभुजाकार मैदान की भुजाएँ क्रमशः 7 मी. , 8 मी. एवं 9मी. है, तो मैदान का अर्द्धपरिमाप ज्ञात करें।

The sides of a triangular field are respectively is 7 m., 8 m. and 9 m. then find the semi-perimeter of the field.

प्रश्न - 06. लघूत्तरीय प्रश्न – **$5 \times 3 = 15$**

Question - 06. Short Answer Type Questions -

1. एक घनाभ की लम्बाई 3 मी, चौड़ाई 2मी., तथा उँचाई 4 मी. है , तो घनाभ का सम्पूर्ण पृष्ठीय क्षेत्रफल ज्ञात करें।

The length of a cuboid is 3 m., breadth 2 m. and height 4 m., then find the total surface area of room.

----

2. आवृत्ति तथा प्रेक्षणों को उदाहरण सहित समझाइए ।

Explain frequency and observations with examples.

3. $6\,\overline{2}\,\overline{4}$ विनकुलम संख्या को सामान्य संख्या मे बदलिए ।

$6\,\overline{2}\,\overline{4}$ Convert Vinkulam number to general number.

----

4. बहुपद से आप क्या समझते हैं उदाहरण देकर समझाए ।

What do you understand by polynomial, explain with example.

----

5. एकाधिकेन पूर्वेण सूत्र के द्वारा निम्न का योगफल ज्ञात कीजिए ।

Find the sum of the following by using the Ekadhiken Purven Sutra.

$$\begin{array}{r} 98765 \\ 34549 \\ +\ 12757 \\ \hline \end{array}$$

प्रश्न - 07. दीर्घ उत्तरीय प्रश्न – **$4 \times 5 = 20$**

Question - 07. Long Answer Type questions -

1. निम्न लिखित आंकडों के लिये दस-दस वर्ग के अन्तराल लेकर बारम्बारता सारणी का निर्माण कीजिए।

   Construct a frequency table for the following data by taking 10 - 10 class intervals.

   13, 11, 8, 19, 0, 44, 27, 10, 8, 35, 13, 27, 31, 18, 42, 23, 19, 34, 19, 27, 43, 17, 7

   ----

2. बौधायन शुल्बसूत्र के निम्न श्लोक की व्याख्या करते हुए, बताइये यह सूत्र कब प्रयोग किया जाता है।

Explaining the following shlok of Baudhayana Shulbasutra, state when this sutra is used?

दीर्घचतुरस्रस्याक्षणया पार्श्वमानी रज्जुः तिर्यङ्मानी

च यत्पृथग्भूते कुरुतस्तदुभयं करोति ।   (बौधायनशुल्बसूत्र 1.48)

----

3. एक त्रिभुज की भुजाएँ 15 मी., 11मी. और 6मी. है तो त्रिभुज का क्षेत्रफल ज्ञात करें।

The sides of a triangle are respectively is 15 m., 11 m. and 6 m. then find the area of the triangle

----

----

4. निम्न श्लोक की व्याख्या कर हल करें। /Solve by interpreting the following shloka.

कोटिश्चतुष्टयं यत्र दोस्त्रयं तत्र का श्रुतिः।
कोटिं दोः कर्णतः कोटिश्रुतिभ्यां च भुजं वद॥

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